RECONSTRUCTION OF INTERFACE BETWEEN TWO ELECTRICALLY CONDUCTING FLUIDS FROM MAGNETIC FIELD MEASUREMENTS

S. Men[1], *C. Resagk*[1], *H. Brauer*[2], *M. Ziolkowski*[2], *M. Kuilekov*[2]

[1] *Department of Mechanical Engineering*
[2] *Department of Electrical Engineering and Information Technology Ilmenau University of Technology, D-98684 Ilmenau, Germany (shouqiang.men@tu-ilmenau.de)*

1. Introduction. Interfacial instability is a general phenomenon in magnetohydrodynamics such as aluminium reduction [1, 2] and other electromagnetic processing of materials [3]. It is hazardous and unwanted in some cases which attract much attention [1]. Among these problems, the interface movement between two electrically conducting fluids is typical and significant in industrial applications.

In this paper, we present a model experiment to study this problem: to measure the displacement of the interface by optical techniques and to reconstruct the interface by measurement of magnetic field caused by an electrical current in the vicinity. In the model, a cylindrical cell containing GaInSn alloy – a liquid metal at room temperature, aqueous solution of KOH and two electrodes is constructed. A periodic vertical vibration of the cell is realized by a pneumatic shaker so as to excite the interface oscillation between the two fluids. In a certain range of amplitude and frequency of vibration, a stable, non-axisymmetric wave pattern is produced, and the velocity and displacement of the oscillating interface are measured by a laser vibrometer. Alternatively, a transparent electrode is fixed at the top surface of KOH solution so that it is possible to observe the wave patterns formed at the interface by a CCD camera from the above.

When a direct current is applied, a magnetic field inside and outside the cell is induced, and two components, the radial and z-components outside, are measured by a sensor-ring consisting of eight two-dimensional fluxgate sensors. It is found, that the measured data processed by fast Fourier transforms, are consistent with the forward calculation. They have been exploited to reconstruct the deformed interface by solving an inverse problem, which is in good agreement with the interface deformation measured by the laser vibrometer.

2. Physical Model. Considering a two-electrically-conducting-fluid system in a cylindrical cell with a radius R, the electrical conductivity of the fluids is k_1 and k_2, respectively. The interface keeps flat in a static state. When an electrical field is applied, the distribution of the current density $\mathbf{J}_0$ is homogeneous if the electrodes at the ends of the cell are large enough or the cylinder is long enough, only azimuthal component of magnetic field can be induced. Whereas the current density $\mathbf{J}$ will become inhomogeneous when the interface is deformed by an exciting force. The maximum deformation of the interface shape can be described as [4]:

$$\eta\left(r,\alpha\right) = A\sum_{m=-M}^{M}\sum_{n=1}^{N}\eta_{mn}J_m\left(k_{mn}r\right)\cdot\cos m\alpha\,, \tag{1}$$

where A denotes the amplitude of the interface, η_{mn} are normalized mode coefficients, n, m is called the radial mode number and the azimuthal number, respectively, J_m is the Bessel function of the first kind, $k_{mn} = y_{mn}/R$ and y_m is the n-th

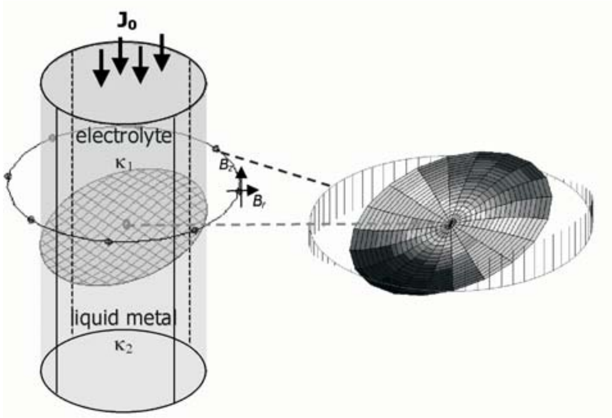

Fig. 1. Mode 11 formed at the interface between two fluids with eight sensors around the cell concentrically, B_z is plotted schematically at the right side.

solution of the equation $J'_m(r) = 0$ for $m > 0$ and the $(n+1)$-th solution at $m = 0$. In addition, the magnetic flux density can be computed by Biot-Savart Law,

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \partial V'. \quad (2)$$

Furthermore, the radial and vertical (z-) components of the magnetic field are only generated once the deformation of the interface is non-axisymmetrical, e.g., mode 11 shown in Fig. 1.

3. Experimental setup. Fig. 2 shows the schematic of the experimental setup. The cylindrical cell is made of plexiglass so that it is insulating and transparent, its radius R is equal to 25mm. GaInSn alloy and aqueous solution of KOH with the same depth of 50mm are used as the two electrically conducting fluids, whose conductivities are respectively 3.64×10^6 S/m and 10 S/m. Eight two-dimensional fluxgate sensors FXM-205 manufactured by Aviatronic Ltd. are mounted concentrically on an electronic board at $R = 41$mm. A Keithley data acquisition board with 32 input channels are utilized to receive signals from the sensors.

The deformation of interface is realized by applying a vertical vibration driven by a pneumatic shaker, the frequency is controlled by a digital-anolog-converter (DAC) conveniently in computer, but the amplitude of the vibration of the shaker is adjusted manually.

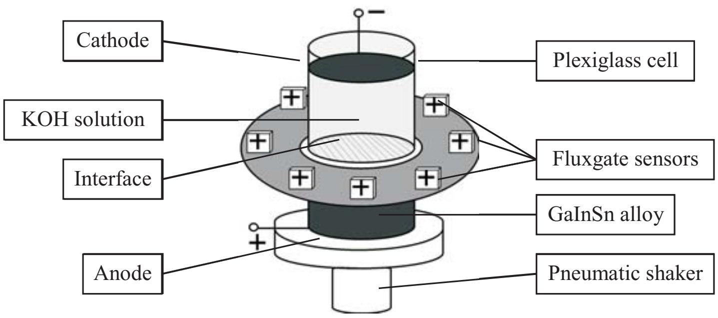

Fig. 2. Schematic of experimental setup.

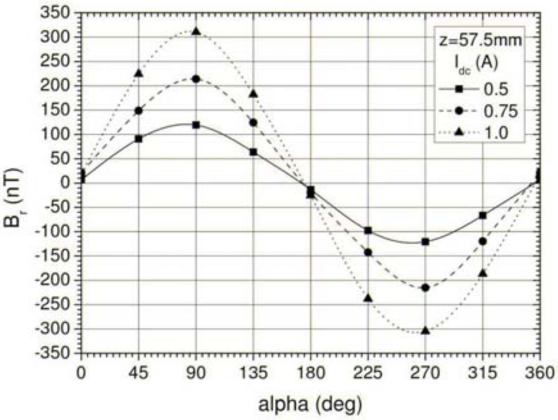

Fig. 3. Amplitude of B_r around the cell at $z = 57.5$mm with current of 0.5A, 0.75A, and 1.0A, the amplitude and frequency of shaker is 1.6mm and 7.5Hz.

4. Results and discussion. When the amplitude and frequency of the shaker is 1.6mm and 7.5Hz, respectively, a stable, periodic wave pattern mode 11 is obtained. The deformation of the interface is approximately 9mm measured by laser vibrometer as well as light sheet techniques, and the oscillating frequency of the interface is 3.75Hz, the half of the driving frequency. It indicates that the interface oscillation is a subharmonic of the driving vibration. When a direct current of 0.5A, 0.75A, and 1.0A is applied alternatively, the radial and z-components of magnetic flux density B_r and B_z are measured by the fluxgate sensors. The amplitude of B_r versus the azimuthal angle alpha in unit of degree at $z = 57.5$mm is plotted in Fig. 3. Since there isn't obvious influence of the electrical current on the interfacial deformation by optical observation, the ratio of the induced B_r should be identical with the ratio of the applied current. The amplitudes of B_r at $I_{\mathrm{dc}} = 1.0$A and 0.75A in Fig. 3 show a good agreement with this prediction. However, there is a little larger discrepancy concerning I_{dc}=0.5A, which is caused by a relatively larger noise-to-signal ratio in case of a smaller electrical current.

By moving the electronic board together with the sensors mounted on it, the amplitudes of the B_r and B_z at different z- positions ranging from 52.5mm to 67.5mm are measured and depicted in Fig. 4. It shows that there are one maximum and one minimum with the identical value for B_r, which is quantitatively consistent with the simulated result in Fig. 5. The distribution of B_z also shows the symmetry versus alpha, and the measured result at position of alpha between 0-180° agrees well with the simulation, too. The only difference lies in the values at positions of alpha=180°–360°. This is mainly originated from the incomplete flat interface in experiments whereas a purely flat interface is used in the simulation.

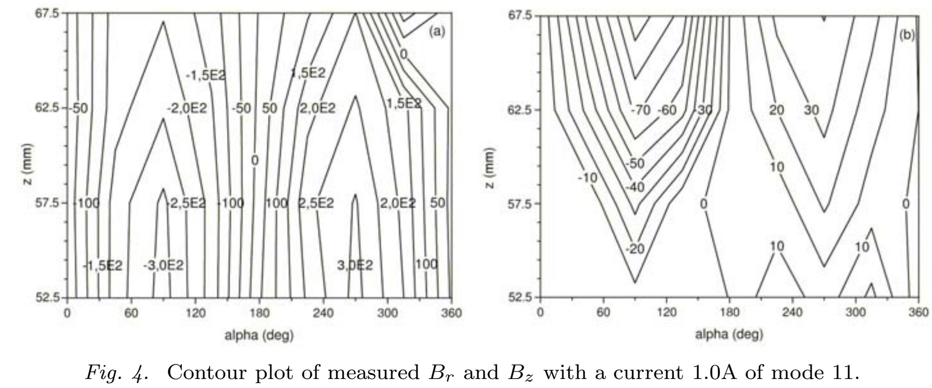

Fig. 4. Contour plot of measured B_r and B_z with a current 1.0A of mode 11.

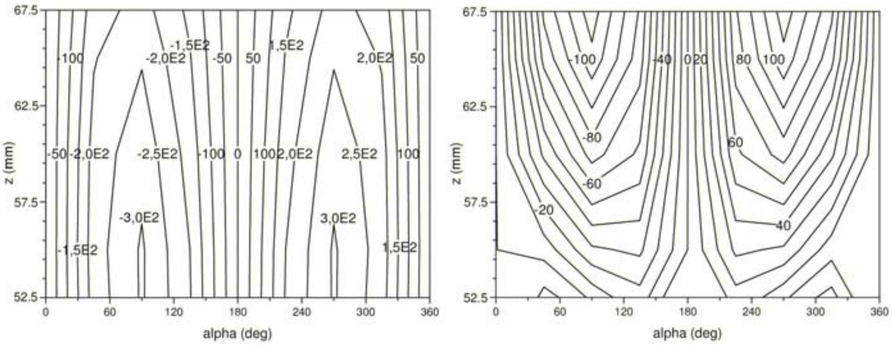

Fig. 5. Contour plot of simulated B_r and B_z with a current 1.0A of mode 11.

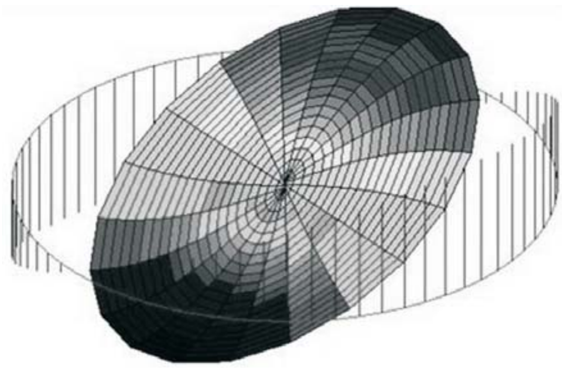

Fig. 6. Reconstructed interface of mode 11 with B_z marked around it.

Based on the measured amplitude of the interface displacement and the radial and z-components of the magnetic flux density, an evolutionary strategy is utilized to reconstruct the deformed interface as shown in Fig. 6. A perfectly flat interface is reconstructed for mode 11 with an amplitude of 9.0mm. The reconstruction error is about 15%, which reveals that the sensor-system is effective to measure the magnetic field near a deformed interface between two electrically conducting fluids.

5. Conclusion. A model experiment of interface reconstruction concerning electrically conducting fluids is carried out. A magnetic sensor system is developed effectively to measure the perturbation of magnetic flux density, the experimental results are in good agreement with the simulation so that they can be used to reconstruct the deformed interface with an acceptable reconstruction error.

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