# EXPERIMENTAL ASPECTS OF CONTACTLESS INDUCTIVE FLOW TOMOGRAPHY

*Th. Gundrum, G. Gerbeth, F. Stefani, M. Xu*

*Forschungszentrum Rossendorf,*
*P.O. Box 510119, D-01314 Dresden, Germany (F.Stefani@fz-rossendorf.de)*

**Introduction.** In many industrial applications, reaching from silicon crystal growth to continuous casting of steel, a non-invasive, contactless measurement technique for the three-dimensional velocity field of the melt flow would be highly desirable. The concerned metallic and semiconducting melts are often characterized by a high electric conductivity. When exposed to an externally appliedmagnetic field, the melt flow produces electrical currents that lead to a modification of the applied field. This field modification, which is measurable in the exterior of the fluid, can be utilized to reconstruct the velocity field. Such a velocity reconstruction has much in common with the well-known magnetoencephalography (MEG), where neuron activity in the brain is inferred from external magnetic field measurements [1]. But in contrast to MEG, there is the freedom to choose different external magnetic fields which allows to get more information about the velocity field. In this paper we report on a first experimental demonstration of such a *contactless inductive flow tomography (CIFT)*. More details can be found in the paper [2].

**1. Theory.** Quite generally, the ratio of the induced field to the applied field is governed by the so-called magnetic Reynolds number $Rm$, defined as $Rm = \mu\sigma v l$, with $\mu$ denoting the magnetic permeability of the melt, $\sigma$ is its electric conductivity, $v$ is a typical velocity,and $l$ is a typical length scale of the flow. Only in very special applications (fast breeders, dynamo experiments), $Rm$ is much larger than 1. In many industrial melt flows, $Rm$ is of the order of 0.01...1. Suppose now that the melt has a stationary velocity field $\mathbf{v}(\mathbf{r})$ and that it is exposed to an externally applied magnetic field $\mathbf{B}_0$. According to the Ohm's law in moving conductors, we get a current $\mathbf{j} = \sigma(\mathbf{v} \times \mathbf{B_0} - \nabla\varphi)$, with $\varphi$ denoting the electric potential. This current, in turn, induces an additional magnetic field $\mathbf{b}$ with

$$\begin{aligned}\mathbf{b}(\mathbf{r}) \quad = \quad & \frac{\mu_0\sigma}{4\pi}\int_V \frac{(\mathbf{v}(\mathbf{r}') \times \mathbf{B}_0(\mathbf{r}')) \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}V' \\ & -\frac{\mu_0\sigma}{4\pi}\int_S \varphi(\mathbf{s}')\mathbf{n}(\mathbf{s}') \times \frac{\mathbf{r}-\mathbf{s}'}{|\mathbf{r}-\mathbf{s}'|^3}\,\mathrm{d}S' \,. \qquad (1)\end{aligned}$$

Equation (1) follows from inserting the expression for the current into the Biot–Savart's law and transforming the volume integral over $\nabla\varphi$ into a surface integral over $\varphi$. In a strict sense, the applied field $\mathbf{B}_0$ under the integral in Eq. (1) must be replaced by the total field $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, but as long as $Rm$ is small, the induced field $\mathbf{b}$ can be neglected there. The electric potential $\varphi$ at the boundary $S$, in turn, has to fulfill the boundary integral equation

$$\begin{aligned}\varphi(\mathbf{s}) \quad = \quad & \frac{1}{2\pi}\int_D \frac{(\mathbf{v}(\mathbf{r}') \times \mathbf{B}_0(\mathbf{r}')) \cdot (\mathbf{s}-\mathbf{r}')}{|\mathbf{s}-\mathbf{r}'|^3}\,\mathrm{d}V' \\ & -\frac{1}{2\pi}\int_S \varphi(\mathbf{s}')\mathbf{n}(\mathbf{s}') \cdot \frac{\mathbf{s}-\mathbf{s}'}{|\mathbf{s}-\mathbf{s}'|^3}\,\mathrm{d}S' \,, \qquad (2)\end{aligned}$$

which follows from taking the divergence of the current and utilizing $\nabla \cdot \mathbf{j} = 0$. Then, the Green's theorem can be applied to the solution of the arising Poisson equation $\Delta\varphi = \nabla \cdot (\mathbf{v} \times \mathbf{B})$, with demanding that the current is purely tangential at the boundary [**?**]. Note that Eq. (2) is the basic formula for the vast area of *electric* inductive flow measurement [3], which is, however, not the subject of the present work.

The main result of the paper [4] was that the velocity structure of the flow can be reconstructed from the external measurement of an appropriate component of the induced magnetic field (e.g., the radial component for a spherically shaped fluid volume) and the electric potential at the fluid boundary, apart from an uncertainty in the radial distribution of the flow. In [5] we had proposed a general method how the electric potential measurement at the fluid boundary can be avoided. The main idea is to apply the external magnetic field in two different, e.g., orthogonal, directions and to measure both corresponding sets of induced magnetic fields. For this purpose, it is needed to treat the electric potential at the boundary in an implicit way. The mathematical details on how to solve the inverse problems can be found in [5, 6, 2].

**2. Experiment.** In the experiment (Fig. 1) we use a polypropylene vessel with 18.0 cm diameter. Approximately 4.4 liters of the eutectic alloy $Ga^{67}In^{20.5}Sn^{12.5}$ (which is liquid at room temperatures) are stirred by a motor driven propeller with a diameter of 6 cm. The propeller is positioned approximately at one third of the total hight, measured from the top. Eight guiding blades above the propeller are intended to remove the swirl of the flow for the case that the propeller pumps upward. In contrast to that, the downward pumping produces, in addition to the main poloidal motion, a considerable toroidal motion. The rotation rate of the propeller can reach up to 2000 rpm, which amounts to a mean velocity of approximately 1m/s, corresponding to a magnetic Reynolds number of approximately 0.4.

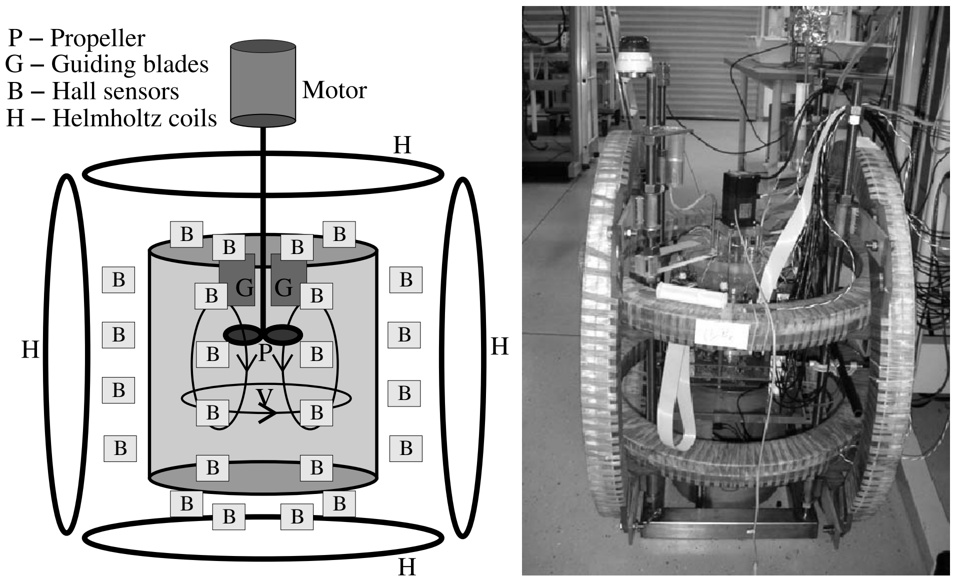


*Fig. 1.* Scheme (*a*) and photograph (*b*) of the CIFT experiment.

The axial and the transverse fields of about 4 mT are alternately produced by two pairs of Helmholtz coils, which are fed by the currents of 22.5 A and 32.5 A, respectively. Both fields are applied for a period of 3 seconds, during which a trapezoidal signal form is used. Taken all together, this allows an online monitoring with a time resolution of 6 seconds. The induced magnetic fields are measured by 48 external Hall sensors, 8 of them grouped together on 6 circuit boards, which are located on different heights (Fig. 1). The most essential problem of the method is the reliable determination of comparably small induced magnetic fields on the background of much higher imposed magnetic fields. In order to achieve this objective, an accurate control of the external magnetic field is necessary. The current drift in the Helmholtz coils is controlled with an accuracy of better than 0.1 per cent. This is sufficient since the measured induced fields are approximately 1 per cent of the applied field. The temperature drift of the Hall sensor's sensitivity can be overcome by enforcing the applied current to be constant. The temperature drift of the Hall sensor's offset is circumvented by changing the sign of the applied

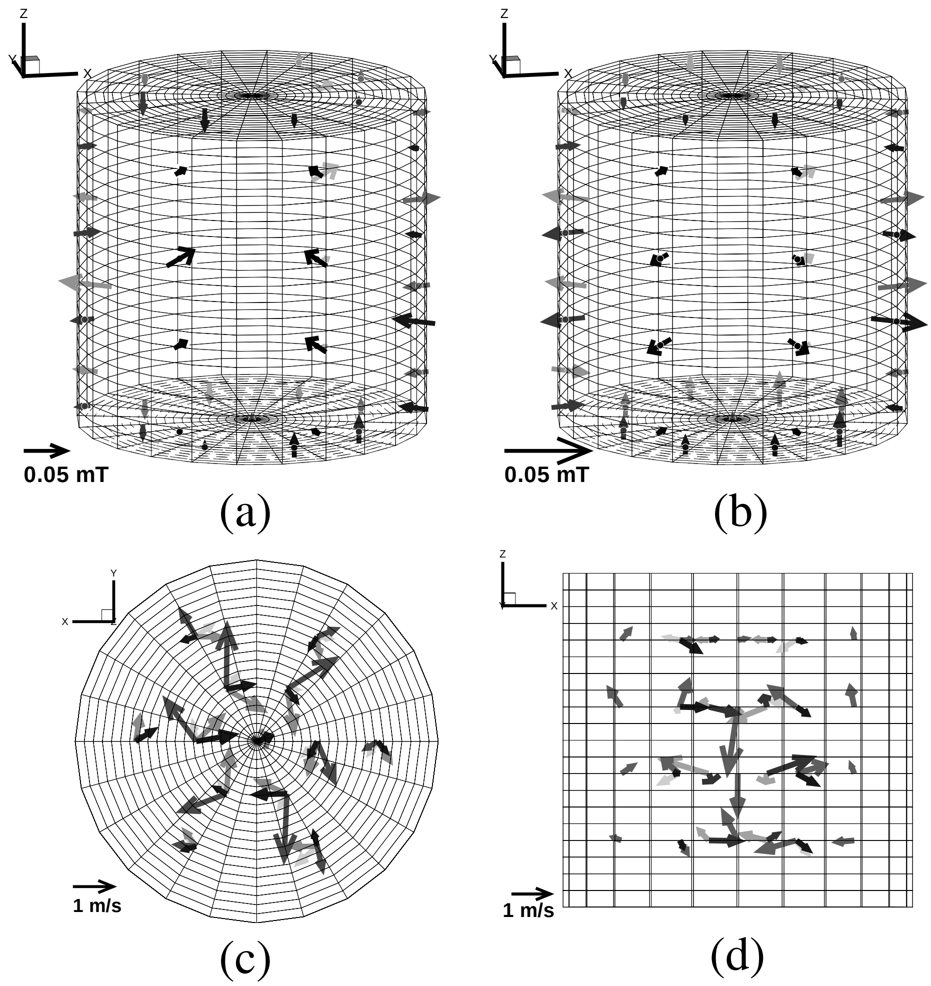


*Fig. 2.* Measured induced magnetic field components for transverse (*a*) and axial (*b*) applied magnetic fields, and reconstructed velocity as seen from below (*c*) and from the side (*d*). The grey scale of the arrows indicates the distance from the eye. The propeller pumps *downward* with 1200 rpm.

magnetic field. For the case that the propeller pumps downward, Fig. 2 shows the induced magnetic fields measured at 48 positions, and the inferred velocity field at 52 discretization points. In Fig. 2*c* we can identify clearly the rotation of the flow, and in Fig. 2*d* the downward flow in the center and the upward flow close to the rim of the vessel. In order to validate the CIFT method, we have performed independent velocity measurements based on ultrasonic Doppler velocimetry (UDV). The comparison with UDV measurements shows that the method provides correct and robust results on the main structure and the amplitude of the velocity field.

**3. Conclusion.** We have demonstrated the feasibility of contactless inductive flow tomography by using two orthogonal imposed magnetic fields. A particular power of CIFT consists in a transient resolution of the full three-dimensional flow structure in steps of several seconds. This enables an online monitoring of slowly changing flow fields in various processes. In our set-up, the externally applied magnetic field is weak and does not influence the flow to be measured. However, CIFT works as well in cases, where stronger magnetic fields are already present for the purpose of flow control, as, e.g., the electromagnetic brake in steel casting or the DC-field components in silicon crystal growth. Obviously, the future of the method lays with replacing the DC fields by AC fields with different frequencies in order to improve the depth resolution of the velocity field and to make the method less sensitive to electromagnetic noise. The integral equation formulation, which can be used for such AC problems, has been developed in [7].

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