ANALYTICAL STUDY OF ELECTROMAGNETIC SHAPING OF LIQUID METAL DROPS IN TRANSIENT MAGNETIC FIELDS

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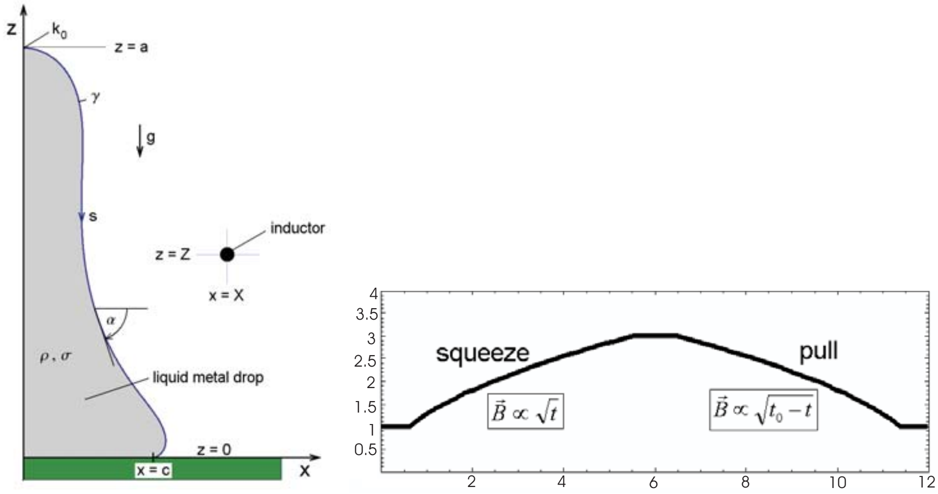
We study analytically the shaping of liquid metal drops using time-dependent magnetic fields. We consider perfectly non-wetting drops of finite curvature that are placed on a horizontal plate. The magnetic fields are generated by an inductor that is fed by an electric current I showing a square root-type time dependence of the form $I \propto t^{1/2}$. In this case the interactions of the applied magnetic field and the eddy currents induced within the liquid metal generate a steady distribution of the Lorentz forces inside the drop. These Lorentz forces can be used to squeeze the drop against the actions of both gravity and surface tension. We derive a non-linear ordinary differential equation for the drop contour describing steady-state deformations. This equation is integrated numerically using a standard shooting method. We show results of our analysis for various values of the Bond number Bo, i.e., drop volumes, and the magnetic Bond number $\mathrm{Bo_M}$, i.e., magnitudes of the Lorentz force.

Introduction. The principle of electromagnetic shaping is commonly used in metallurgical applications [1]. Usually, a high-frequency magnetic field is imposed on a liquid metal with a free surface. Inside the metal, therefore, Lorentz forces $\mathbf{f}_\mathrm{L} = \mathbf{J} \times \mathbf{B}$ will arise. The high frequency leads to a skin-effect limiting the effects of the Lorentz forces to the vicinity of the surface. These forces basically act as a magnetic pressure consisting of a constant and an oscillating part. The constant part allows to squeeze and to support liquid metal surfaces. Alternately, the oscillating part presses and pulls on the metal surface thus bearing the potential of causing instabilities, see Kocourek et al. [2] and Conrath & Karcher [3], who studied the behaviour of liquid metal drops in high-frequency magnetic fields. For low-frequency magnetic fields a similar approach was performed in the "Starfish experiment" of Sneyd et al. [4].

Our arrangement consists of a longitudinal liquid metal drop submitted to a monotonically increasing magnetic field. The field is generated by an inductor fed by an electric current of the form $I \propto t^{1/2}$. In this case, the Lorentz forces are steady and fully penetrate the drop. They tend to push the drop away from the inductor. Analogously, for a monotonically decaying inductor current, i.e., $I \propto (t_0 - t)^{1/2}$, the Lorentz forces act towards the inductor, i.e., pulling the drop.

The results presented here are restricted to the case of steady squeezing of the drop. We analyze the effects of drop volume and magnetic field strength on the squeezing. The paper is organized as follows. In Sec. 1 we present the mathematical model. In Sec. 2 we discuss the main results of this model. Finally, Sec. 3 provides a summary.

1. Mathematical model. Fig. 1*a* shows the geometry of the analyzed arrangement. To squeeze the drop with a constant force, the inductor is fed by an electric current $I = I_0 \cdot \sqrt{1+2Ft}$. Then the magnetic field will be of the form $\mathbf{B} = \mathbf{B}_0(x,z) \cdot \sqrt{1+2Ft}$ too, see Fig. 1*b*. Here, F is a constant factor. Applying a Faraday's law, the induced eddy currents take the form $\mathbf{J} = \mathbf{J}_0(x,z) \cdot F/\sqrt{1+2Ft}$. Finally, for the induced Lorentz forces we obtain $\mathbf{f}_L = \mathbf{f}_{L0}(x,z) \cdot F$. In detail, the



(*a*) (*b*)

Fig. 1. (*a*) Geometry and properties. (*b*) Function of the magnetic field in time.

Lorentz forces write as follows:

$$\mathbf{f}_L = \left\{ \begin{array}{c} -F\cdot\frac{\sigma}{2}\cdot\left(\frac{\mu I_0}{2\pi}\right)^2\cdot\ln\left[\frac{(x-X)^2+(z-Z)^2}{(x+X)^2+(z-Z)^2}\right]\cdot\left[\frac{x-X}{(x-X)^2+(z-Z)^2}-\frac{x+X}{(x+X)^2+(z-Z)^2}\right] \\ - \\ -F\cdot\frac{\sigma}{2}\cdot\left(\frac{\mu I_0}{2\pi}\right)^2\cdot\ln\left[\frac{(x-X)^2+(z-Z)^2}{(x+X)^2+(z-Z)^2}\right]\cdot\left[\frac{z-Z}{(x-X)^2+(z-Z)^2}-\frac{z-Z}{(x+X)^2+(z-Z)^2}\right] \end{array} \right\} \cdot \left\{ \begin{array}{c} \mathbf{e}_x \\ 0 \\ \mathbf{e}_z \end{array} \right\}. \tag{1}$$

In this equation, x and z denote the spatial coordinates inside the drop, whereas X and Z mark the position of the inductor, respectively, cf. Fig 1*a*. Obviously, these Lorentz forces are a function of space but not of time. Likewise, as the magnetic field is decreased, we obtain a constant force in the opposite direction, cf. Fig. 1*b*. The integration of the Lorentz forces between $x = 0$, $z = 0$ and the drop surface yields the static pressure distribution

$$p_{hs}(x,z) = -F\cdot\frac{\sigma}{2}\cdot\left(\frac{\mu I_0}{4\pi}\right)^2\cdot\left\{\ln\left[\frac{(x-X)^2+(z-Z)^2}{(x+X)^2+(z-Z)^2}\right]\right\}^2 - \rho g z + p_0\,. \tag{2}$$

Here, σ, μ, ρ and p_0 denote the electrical conductivity, the magnetic permeability, the fluid density and the bottom pressure at $z = 0$, respectively. Once the magnetically induced pressure on the surface is known, we can insert it into the so-called Young–Laplace equation which describes the pressure equilibrium on the drop surface. This equation reads as

$$\gamma\cdot k(x,z) = F\cdot\frac{\sigma}{2}\cdot\left(\frac{\mu I_0}{4\pi}\right)^2\cdot\left\{\ln\left[\frac{(x-X)^2+(z-Z)^2}{(x+X)^2+(z-Z)^2}\right]\right\}^2 + \rho g(z-a) + \gamma\cdot k_0\,, \tag{3}$$

where γ is the surface tension of the liquid, $k(x,z)$ is the curvature on the drop surface, and k_0 is the surface curvature at $z = a$, cf. Fig. 1*b*. For a further treatment of the problem it is convenient to introduce dimensionless numbers. Therefore, we choose the drop height a as a characteristic dimension and define the Bond number Bo and the electromagnetic Bond number $\mathrm{Bo_M}$ as follows. These parameters relate the hydrostatic pressure and the magnetically induced pressure to the surface tension pressure, respectively.

$$\mathrm{Bo} = \frac{\rho g a^2}{\gamma}\,, \qquad \mathrm{Bo}_\mathrm{M} = F \cdot \frac{\sigma}{2} \cdot \left(\frac{\mu I_0}{4\pi}\right)^2 \cdot \frac{a}{\gamma}\,. \tag{4}$$

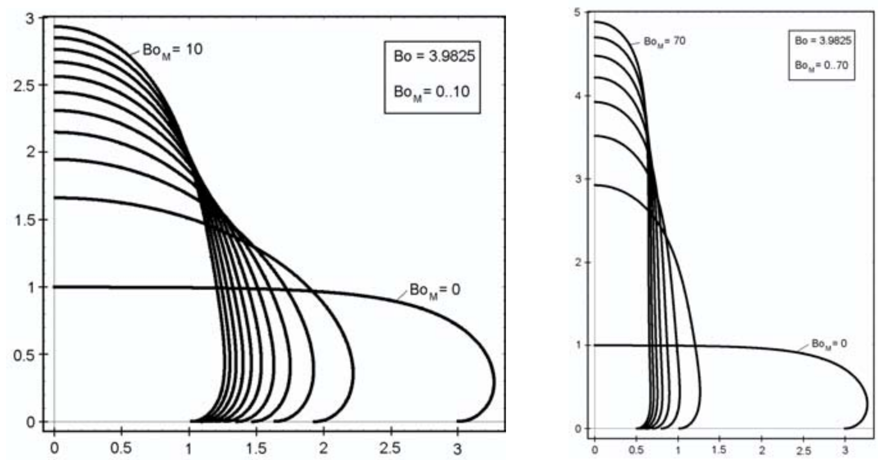
The normalized Young–Laplace equation now reads as

$$k\,(x,z) = \kappa_M \cdot f_M\,(x,z) + \mathrm{Bo} \cdot (z-1) + k_0 \tag{5}$$

where $f_\mathrm{M}(x,z) = \left\{\ln\left[\frac{x-X)^2+(z-Z)^2}{(x+X)^2+(z-Z)^2}\right]\right\}^2$. Since the curvature is a nonlinear term of the form $k(x,z) = z''/(1+z'^2)^{3/2}$, Eq. (5) cannot be solved analytically. Therefore, we apply a shooting method based on a numerically aided geometrical construction of the drop contour. We rewrite the curvature as $k(x,z) = \mathrm{d}\alpha/\mathrm{d}s$, where α and s are the surface angle and arc length, cf. Fig. 1*b*. Starting from the top of the drop at $x = 0$ and $z = a$, we choose a special initial angle $\alpha = 0$ and an arbitrary initial curvature $\mathrm{d}\alpha/\mathrm{d}s$. We imply an improved Euler method [5] to calculate point after point of that contour. This procedure is stopped when the contact angle of $\alpha_\mathrm{C} = 180°$ is attained. As we do not know the initial curvature beforehand, after each shot we have to adjust it iteratively until the final height of the point $\alpha = 180°$ becomes zero.

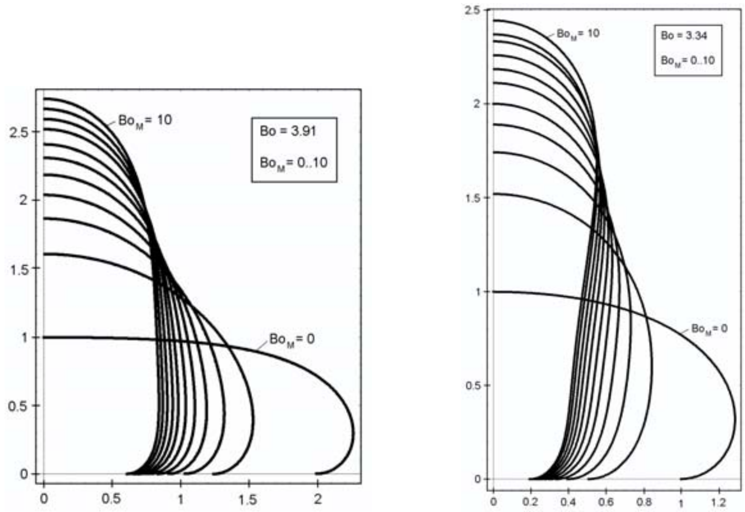
2. Results and discussion. Fig. 2 shows the results for a drop that originally spreads to $c_0 = 3$. The inductor is located at $X = 5$ and $Z = 0$. As the magnetic field increases, the drop is squeezed. During the squeezing process the drop volume is conserved at its initial value, where $\mathrm{Bo}_\mathrm{M} = 0$. Fig. 2*a* shows the resulting drop contours for a stepwise increase of $\mathrm{Bo}_\mathrm{M} = 0 \ldots 10$. In the case of $\mathrm{Bo}_\mathrm{M} = 10$ the drop height is nearly tripled, while the contact position is reduced by a factor of three. Fig. 2*b* shows the squeezing of the same drop in the range $\mathrm{Bo}_\mathrm{M} = 0 \ldots 70$. The drop mounts up to about five times its original height, while the contact position is reduced by a factor of six.

Fig. 3*a* shows the squeezing of a drop that initially spreads to $c_0 = 2$. Here, the inductor is located at $X = 10/3$ and $Z = 0$ to keep the relative position between the inductor and the contact point of the drop. Due to the smaller Bond number, the effect of gravity diminishes. As a result, in the case of $\mathrm{Bo}_\mathrm{M} = 10$ the slope of the drop flank is apparently steeper compared to Fig. 2. Beside, the



(*a*) (*b*)

Fig. 2. Inductor at $\{X, Z\} = \{5, 0\}$, (*a*) $\mathrm{Bo}_\mathrm{M} = 0 \ldots 10$, (*b*) $\mathrm{Bo}_\mathrm{M} = 0 \ldots 70$.



(*a*) (*b*)

Fig. 3. Inductor at (*a*) $\{X, Z\} = \{10/3, 0\}$ and at (*b*) $\{X, Z\} = \{5/3, 0\}$.

contact position is divided by a factor close to four. Fig. 3*b* shows the squeezing of a drop that initially spreads to $c_0 = 1$. Here, the inductor is located at $X = 5/3$ and $Z = 0$. In the case for $\mathrm{Bo_M} = 10$, the drop even hangs over and the contact position is divided by a factor of five.

3. Summary. We have investigated analytically the steady electromagnetic squeezing of a liquid metal drop. We use a special time-dependence of the magnetic field to achieve steady electromagnetic forces. The resulting Young–Laplace equation describing the drop surface is solved by a shooting method. We examine the effect of the magnetic field strength and drop volume on the squeezing. According to our results, this kind of magnetic field seems highly suitable for electromagnetic shaping purposes.

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