NOVEL POTENTIALITIES OF ELECTROMAGNETIC STIRRING OF MELTS IN METALLURGY

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The efficiency of technological processes of producing metals and alloys, continuous ingots and castings of ferrous and non-ferrous metals is mainly determined by the intensity of heat and mass transfer in the liquid phase.

The Electromagnetic stirring (EMS) method developed and proposed by Energetics Technologies Ltd (ET) provides a possibility of controlling such heat and mass transfer. It uses anharmonic traveling (rotating) magnetic fields excited by a system of electric currents whose amplitude and frequency are modulated by periodic (in time) functions with the frequency exceeding the carrier frequency. With a suitable choice of modulation parameters, we can considerably increase the intensity of melt stirring at the expense of increased turbulent transfer intensity due to the excitation of the so-called forced turbulence, without increasing the mean velocity of convective flows. Thus, in this case, a more intense mixing can be achieved due to a more intense turbulent transfer at a reduced mean velocity (convective transfer).

1. Estimation of mean velocity of turbulent flows. Phenomena arising in liquid metals turbulently rotating under the action of modulated RMF are studied in an induction-free approximation using the "external" friction model [1].

In vessels of circular cross-section, these phenomena are described in a cylindrical coordinate system r, ϕ, z rotating at an angular velocity $\bar{\Omega}$ ($\bar{\Omega} = \Omega/\omega_0$ is a dimensionless angular velocity of the rotation of quasi-solid turbulent flow core) by the following system of dimensionless equations:

$$\frac{\partial^2 V_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r^2} - \lambda V_\varphi = -\mathrm{Ha}^2\left(S_1 r + \frac{e^2}{2} S_2 r^3\right), \quad (1)$$

where $S_1 = 1 - \Omega$; $S_2 = (k+1)/2 - \Omega$; $\Omega = V_\varphi / r$.

In vessels of rectangular cross-section, these phenomena are described by the following equations in Cartesian coordinates x, y, z:

$$\Delta V_x - \lambda V_x = \mathrm{Ha}^2\left\{y\left[1 + \frac{e^2(k+1)}{4}\left(x^2+y^2\right)\right] + V_x\left[1 + \frac{e^2}{12}\left(a^2+b^2\right)\right]\right\}, \quad (2)$$

$$\Delta V_y - \lambda V_y = -\mathrm{Ha}^2\left\{x\left[1 + \frac{e^2(k+1)}{4}\left(x^2+y^2\right)\right] - V_y\left[1 + \frac{e^2}{12}\left(a^2+b^2\right)\right]\right\}, \quad (3)$$

$$V_\Gamma = 0, \quad (4)$$

where $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$; e is the amplitude modulation depth, k is the ratio of modulation frequency to the carrier frequency, $\mathrm{Ha} = B_0 R_H \sqrt{\sigma/\eta}$, $\lambda = C_\varepsilon \left(\mathrm{Re}_\omega < \Omega >\right)^{1-\varepsilon}/\delta_z$, $\delta_z = Z_0/R_H$, $R_H = R_0 = 2ab/(a+b)$,

$$\langle\Omega\rangle = \frac{1}{2}\int_0^1 |\mathrm{rot}\mathbf{v}|\mathrm{d}r = \frac{1}{2ab}\int_0^a\int_0^b |\mathrm{rot}\mathbf{v}|\mathrm{d}x\mathrm{d}y.$$

Expressions for electromagnetic body forces (EMBF) are derived basing on the solution of the equation for the z-component of the vectorial potential of magnetic induction ($\mathbf{b} = \operatorname{rot}\mathbf{a}$):

$$\frac{\varpi}{2\pi}\left(\frac{\partial a_z}{\partial \tau} + \Omega\frac{\partial a_z}{\partial \varphi}\right) = \Delta a_z \tag{5}$$

with the boundary condition

$$\left.\frac{\partial a_z}{\partial r}\right|_{r=1} = -\left(1 + e\sin 2\pi\phi_M\right)\sin 2\pi\phi_0, \tag{6}$$

where $\phi_M = k\tau - \varphi$, $\phi_0 = \tau - \varphi$, $\varpi = \mu_0\sigma\omega R_H^2$.

The solution of Eqs. (1)–(4) has the form

$$V_\varphi = \mathrm{Ha}^2\left\{\frac{1}{\beta_1^2}\left[r - \frac{I_1(\beta_1 r)}{I_1(\beta_1)}\right] + \sum_{n=1}^{\infty} V_{2n} I_1(\alpha_n r)\right\}, \tag{7}$$

where $\beta_1^2 = \lambda + \mathrm{Ha}^2$, $V_{2n} = \dfrac{\mathrm{Ha}^2 e^2}{2}(k+1)\displaystyle\int_0^1 r^4 J_1(\alpha_n r)\mathrm{d}r / J_0^2(\alpha_n)$; α_n are the roots of equation $J_1(\alpha_n) = 0$.

$$V_x = \sum_{n=0}^{\infty}\left[P_{1n}(a)\frac{\mathrm{ch}\beta_{1n}x}{\mathrm{ch}\beta_{1n}a} - P_{1n}(x)\right]\cos\nu_n y + \left[P_{2n}(b)\frac{\mathrm{sh}\beta_{2n}y}{\mathrm{sh}\beta_{2n}b} - P_{2n}(y)\right]\cos\mu_n x, \tag{8}$$

$$V_y = \sum_{n=1}^{\infty}\left[P_{1n}^*(x) - P_{1n}^*(a)\frac{\mathrm{sh}\beta_{1n}x}{\mathrm{sh}\beta_{1n}a}\right]\cos\nu_n y + \left[P_{2n}^*(y) - P_{2n}^*(b)\frac{\mathrm{ch}\beta_{2n}y}{\mathrm{ch}\beta_{2n}b}\right]\cos\mu_n x, \tag{9}$$

where $\nu_n = \pi\left(n - 1/2\right)/b$; $\mu_n = \pi\left(n - 1/2\right)/a$; $\beta_{1n}^2 = \nu_n^2 + \lambda + \mathrm{Ha}^2 C$;

$$\beta_{2n}^2 = \mu_n^2 + \lambda + \mathrm{Ha}^2 C; \qquad C = 1 + e^2(a^2 + b^2)/12;$$

$$P_{1n}(x) = \frac{\mathrm{Ha}^2}{2b\beta_{1n}^2}\left\{\frac{b}{\nu_n}(-1)^n - \frac{1}{\nu_n^2} + \frac{e^2(k+1)}{4}\left[\frac{6}{\nu_n^4} + \left(b^2 - \frac{6}{\nu_n^2}\right)\frac{b}{\nu_n}(-1)^n + x^2\right]\right\},$$

$$P_{2n}^*(y) = \frac{\mathrm{Ha}^2}{2a\beta_{2n}^2}\left\{\frac{a}{\mu_n}(-1)^n - \frac{1}{\mu_n^2} + \frac{e^2(k+1)}{4}\left[\frac{6}{\mu_n^4} + \left(a^2 - \frac{6}{\mu_n^2}\right)\frac{a}{\mu_n}(-1)^n + y^2\right]\right\},$$

$$P_{1n}^*(x) = \frac{\mathrm{Ha}^2(-1)^n x}{b\nu_n\beta_{1n}^2}\left\{1 + \frac{e^2(k+1)}{4}\left(b^2 - \frac{2}{\nu_n^2} + \frac{6}{\beta_{1n}^2} + x^2\right)\right\},$$

$$P_{2n}(y) = \frac{\mathrm{Ha}^2(-1)^n y}{a\mu_n\beta_{2n}^2}\left\{1 + \frac{e^2(k+1)}{4}\left(a^2 - \frac{2}{\mu_n^2} + \frac{6}{\beta_{2n}^2} + y^2\right)\right\}.$$

To determine the initial angular velocity value involved in the expression for λ, the following algebraic equation derived from Eq (1) at $V_\varphi = \Omega r$ is used:

$$\Omega^{2-\varepsilon} + Q_\varepsilon\left(1 + e^2/3\right)\Omega - Q_\varepsilon\left(1 + \frac{e^2}{6}(k+1)\right) = 0, \tag{10}$$

where $Q_\varepsilon = \mathrm{Ha}^2\delta_z/\mathrm{Re}_\omega^{1-\varepsilon}C_\varepsilon$.

Computations have shown that for low values of the Hartmann number, the velocity profile looks as a cubic parabola, while for high values it acquires the form of a quasi-solid core of the near-wall and boundary layer, whose thickness decreases with the growing Hartmann number.

2. Specrtal density of turbulence in modulated and non-modulated cases. It is well-known that the rotation of a conducting fluid under the action of an external rotating magnetic field is turbulent. Spectral density of turbulent energy in the inertial interval depends on the frequency of turbulent fluctuations mode ω as follows:

$$E(\omega) \sim \omega^{-\gamma}\,, \tag{11}$$

where γ is a spectral exponent in the inertial interval.

If energy is supplied to turbulence only from large-scale vortices with the frequency ω_0 and then is redistributed over vortices of various sizes, the energy of vortices with the frequency ω can be determined as

$$E(\omega) \sim E_0(\omega_0)\left(\frac{\omega_0}{\omega}\right)^{(\gamma+1)}\,, \tag{12}$$

where $E_0(\omega_0)$ is the energy injected into turbulence. It is noteworthy that the energy $E_0(\omega_0)$ can be introduced into turbulent fluctuations both through the mean flow instability (i.e., it can be connected with the mean velocity value, – case 1) and as a result of fluctuations of the force connected with the external RMF fluctuations at frequencies close to ω_0 (case 2). If the part of mean flow energy transferred to turbulent fluctuations does not exceed 10–15%, the direct energy injection at frequencies $\sim \omega_0$ is much higher. Hence, if the purpose of the impact is to generate a high turbulence intensity, then it is necessary to form a magnetic field spectrum with fluctuations of the force with the frequency $\sim \omega_0$. In this case, the necessary consumed energy is reduced in comparison with case 1, which is observed experimentally. Naturally, case 3 is also possible, which is a combination of cases 1 and 2. In this case, at the application of a modulated magnetic field, an extensive spectrum of frequencies of the external force fluctuations arises. Such situation is characteristic of the case, where the energy of mean and turbulent motion of liquid metal is due to the work of EMBF arising under the action of RMF.

When turbulent energy is supplied simultaneously to vortices of various scales (and, surely, of various frequencies), the exponent γ in Eq. (12) can change. In fact, let the force $F_0(\omega_0)$ generate large-scale turbulent vortices with the frequency ω_0 in the most general case 3. Then we can evaluate the energy of these vortices as

$$E_0 \sim \alpha_1 (F_0/\omega_0)^2 \tag{13}$$

On the other hand, if the force $F(\omega)$ generates vortices with the frequency $\omega > \omega_0$,

$$E'(\omega) \sim \alpha_2 [F(\omega)/\omega]^2 + E_0\left(\frac{\omega_0}{\omega}\right)^{\gamma}\,. \tag{14}$$

Then in a stationary mode, the final energy distribution in frequencies corresponds to Eq. (12), but with different γ_1 values. Therefore, if

$$E'(\omega) \sim E_0(\omega_0)\left(\frac{\omega_0}{\omega}\right)^{\gamma_1}\,, \tag{15}$$

then, taking into account Eqs. (13) and (14), we obtain that

$$\left(\frac{\omega_0}{\omega}\right)^{\gamma_1} - \left(\frac{\omega_0}{\omega}\right)^{\gamma} = \frac{\alpha_2}{\alpha_1}\left(\frac{F(\omega)}{F_0}\right)^2\,. \tag{16}$$

At $F(\omega) = 0$ we obtain $\gamma = \gamma_1$. It follows from Eq. (16) that since its right-hand part > 0 and $\omega_0/\omega < 1$, $\gamma_1 < \gamma$ is always valid, i.e., with increasing $F(\omega)$ the slope exponent in Eq. (15) decreases. On the other hand, as shown in [4], in scales close to the characteristic scale and exceeding it, the parameter $\gamma < 0$. However, at the same time $\omega < \omega_0$, which results in the growth of $F(\omega)$ and energy growth at the frequencies below ω_0. It means that relative energy of vortices grows with the frequency ω. Assuming that the energy is proportional to the number of vortices N_ω with the specified frequency, we can conclude that N_ω grows, which should essentially affect the properties of local mixing of a liquid metal.

3. Experimental results. Experimental check of the developed estimation methods was performed in vessels of circular or square cross-section made of non-magnetic steel or plastic arranged in the explicit-pole inductor bore. The latter was fed by a programmable three-phase power sources produced by Pacific Power Source (AMX-Series) and California Instruments (iX-Series). They provided various amplitude- and frequency-modulated voltages with modulation depth and frequency varying in a broad range.

The experiments were carried out at room temperature on a ternary eutectic alloy InGaSn and mercury. Magnetic field spatial distribution and spectral characteristics were measured with a teslameter FW Bell 6010 and oscilloscope Fluke 199C. Mean velocities of the turbulent flow were measured using a propeller installed in the quasi-solid flow core and a digital tachometer Line Seiki E90-103. Local velocity values were measured using two-electrode conductive probes with permanent magnets [2, 3] connected to a low-noise preamplifier AMETEK 5113. The signal was processed using an A/D converter PCI 6052E and a virtual measuring system on the basis of LabVIEW 7.0 allowing spectral analysis of dynamic flow characteristics. Mean velocity profiles were also measured by an ultrasonic Doppler velocimeter DOP-2000.

The obtained experimental results compared with the computed ones point to a universal character of the described model.

4. Summary. The proposed method of magnetohydrodynamic (MHD) stirring of melts consists in the use of *anharmonic* traveling (rotating) magnetic fields excited by an m-phase system of amplitude- and frequency-modulated currents. This method allows individual flexible control of the intensity of convective and turbulent heat and mass transfer. We present the results of theoretical studies of MHD effects arising at the usage of modulated rotating magnetic fields in liquid metals. Besides, we present new results that were obtained in laboratory experiments on low-temperature melts.

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