OSCILLATIONS OF LIQUID METAL DROPS IN A HIGH-FREQUENCY MAGNETIC FIELD

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Introduction. We study experimentally the effects of a high-frequency magnetic field on the dynamical behavior of a liquid metal drop. In first model experiments, Kocourek *et al.* [1] and [2] studied the static deformation of a circular liquid metal drop due to an electromagnetic pressure that is generated by a high-frequency magnetic field. They use Galinstan as a test liquid. This metal shows a melting temperature of $-19°$C allowing precise measurements at room temperature. The authors demonstrate that high-frequency fields result in a stable squeezing of the drop. Initially flat drops (due to gravity) show a nearly hemispherical shape when an inductor current of 20 kHz frequency is switched on. This static deformation has also been predicted theoretically by Conrath and Karcher [4]. In their analysis they use a simple model based on both a skin depth approximation and a flat drop approximation. Within these limits they derive a linearized Young–Laplace equation describing the drop contour. This equation can be solved analytically using a Green's function technique.

The purpose of this paper is to study experimentally the instability of the drop in more detail including the evaluation of the oscillation frequencies. For this purpose, we vary the control parameter drop volume V and present an image processing method to evaluate the experimental data. The paper is organized as follows: In Sec. 1 we describe the experimental methods. Some experimental results are shown in Sec. 2. In Sec. 3 we conclude the experimental findings.

1. Experimental methods. In the experiments we use the same test facility as described in Refs. [1, 2, 3], see Fig. 1. The inductor is fed with an alternating electric current with frequency $f = 20$ kHz. We increase the feeding current

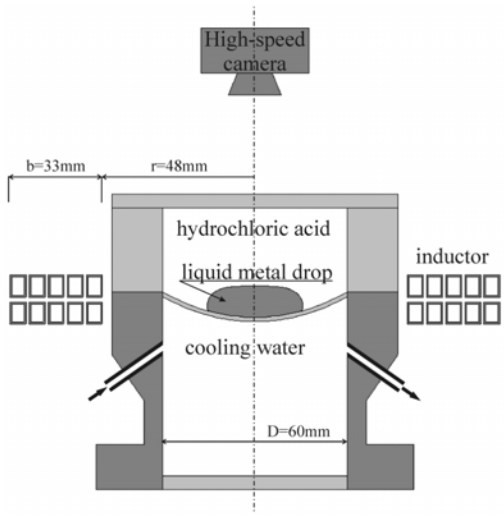

Fig. 1. Sketch of the experimental set-up.

up to 260 A and vary the drop volume within the range of 0.2 ml< V <11 ml. During the experiments we observe the drop shape from above using a high-speed camera system. In the experiment we put the liquid metal into the box, cover the drop fully with hydrochloric acid to avoid oxidation and close the box. The inductor current is changed from 0 A to 260 A in steps between 2.5 A and 20 A. At the beginning we switched on the power supply, set up the required inductor current and switched off the power supply. After cooling down the metal, we again switched on the generator. The required current feeds inductor at once. In 10 s we started the high-speed camera at a rate of 154 fps. Recording length ($t = 6,6495$ s) is defined by the size of the internal camera memory (512 MB). At the end of each experiment we saved the movie in a computer as a sequence of bmp pictures. For analysis of the picture it is very important to achieve a good contrast between the inside and outside drop area. Using lighting from below, we do not observe any reflection on the surface. The liquid metal drop is black and the surroundings white. Each picture taken by the camera is subject to an image processing method using the software MATLAB. For more details see [3].

2. Results. Hydrochloric acid in the experiment changed the effective density of the metal. Using of the Bond number, which is defined as

$$\mathrm{Bo} = f_\mathrm{g}/f_\sigma = \rho \cdot g \cdot V^{2/3}/\sigma_\mathrm{S}, \tag{1}$$

where f_g denotes the gravity force and f_σ is the surface tension force, allows us to compare results from different experiments.

The effect of the electromagnetic field we describe by the electromagnetic Bond number

$$\mathrm{Bo_M} = f_\mathrm{L}/f_\sigma = \frac{\sigma \cdot f \cdot \mu^2 \cdot (N \cdot I)^2 \cdot V^{1/3}}{\sigma_\mathrm{S}}, \tag{2}$$

where f_L is the Lorentz force.

2.1. Bifurcation diagram. Using data from the image processing, we obtain bifurcation diagrams as shown in Fig. 2. Here the amplitudes of the oscillations modes A/R_0 are plotted against the applied magnetic field $\mathrm{Bo_M}$. Again, the current frequency is fixed at $f = 20$ kHz. One example is depicted. In this case (Bo=13.7) the dominant oscillation mode is characterizes by $m = 2$. The

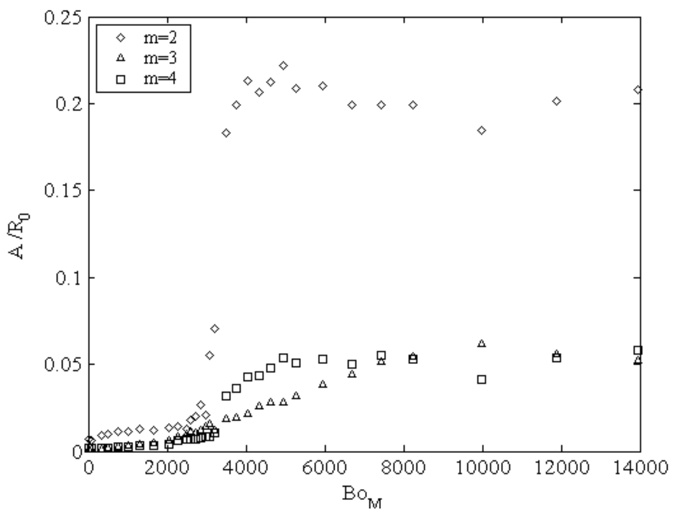

Fig. 2. Bifurcation diagram for the Bond number Bo = 13.7.

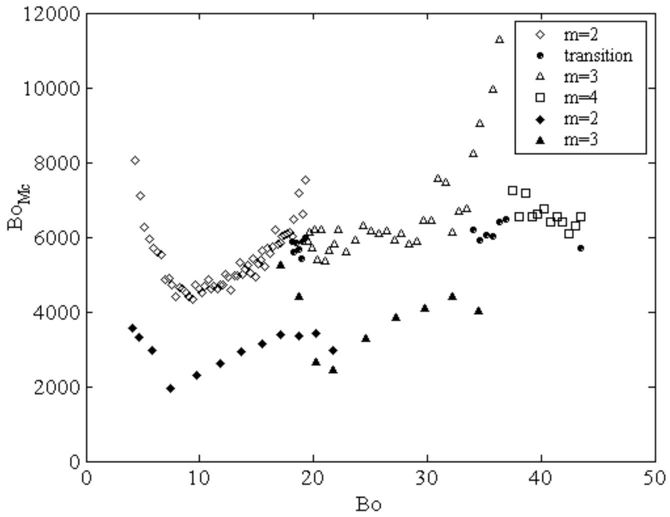

Fig. 3. Stability diagram of liquid metal drops submitted to an electromagnetic field of $f =$ 20 kHz.

amplitude of this mode grows quickly when the electromagnetic Bond number exceeds the critical value of about $Bo_{Mc} = 2900$. We find out this value as follows: at the place $d(A/R_0)/d(Bo_M) = \max$ we define a line, which approximates the 5 nearest measured values. Intersection of the line with the Bo_M-axis gives the critical value. All other modes show a much smaller amplitude.

2.2. Stability diagram. Fig. 3 shows the stability diagram of liquid metal drops submitted to an electromagnetic field of frequency $f = 20$ kHz. Here the critical electromagnetic Bond number Bo_{Mc} is plotted against the Bond number Bo. Beneath the curves, i.e., $Bo_M < Bo_{Mc}$, we observe stable circular drops squeezed by the induced electromagnetic pressure. Above the curves, i.e., $Bo_M > Bo_{Mc}$, these static states are unstable and waves of a particular mode number m and oscillation frequencies f_C are observed. Upon increasing the Bond number

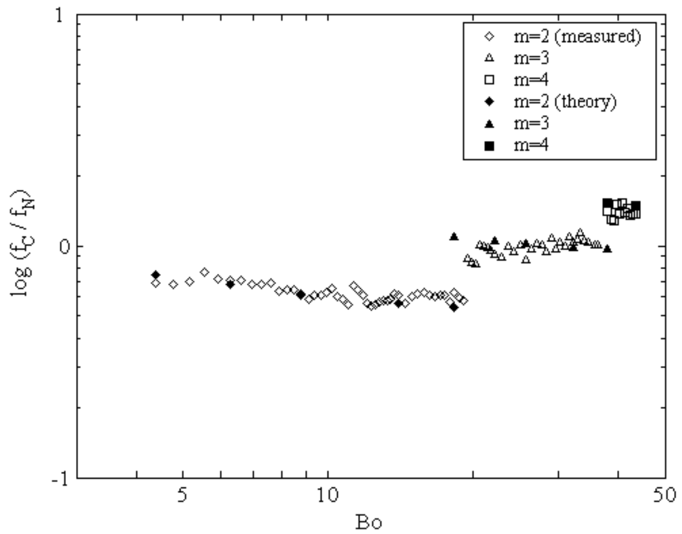

Fig. 4. Normalized critical oscillation frequency f_C/f_N of the various modes as a function of the Bond number Bo.

Bo the first unstable mode is characterized by $m = 2$. This mode corresponds to drop oscillations of a cigar-type shape. This mode is present in the range of $4 < \mathrm{Bo} < 17.5$.

At small Bond numbers, the critical electromagnetic Bond number $\mathrm{Bo}_{\mathrm{Mc}}$ decreases rapidly with the increasing Bo, but at higher values of *Bo* starts to increase. In this range a transition to mode number $m = 3$ with triangular symmetry takes place. This scenario is repeated at a Bond number of about $\mathrm{Bo} = 34.5$, where the transition from $m = 3$ to $m = 4$ occurs, i.e., the transition from modes with triangular symmetry to modes with square symmetry.

2.3. Critical frequency. Using the image processing method we evaluate the critical frequencies f_{C} of the drop oscillations. The results are shown in Fig. 4 for the inductor current frequency of $f = 20$ kHz. Here we have plotted the normalized critical oscillation frequency $f_{\mathrm{C}}/f_{\mathrm{N}}$, where $f_{\mathrm{N}} = \sqrt{\rho \cdot V/\sigma_{\mathrm{S}}}$, for the observed mode numbers $m = 2$, 3, and 4, respectively, as a function of the Bond number. The hollow symbols denote the experimental data, while the full symbols represent the theoretical predictions according to the relation

$$f_{\mathrm{C}} = \frac{1}{2\pi} \left(\frac{\sigma_{\mathrm{S}} m \left(m^2 - 1\right)}{\rho R^3} \right)^{1/2} . \tag{3}$$

In Eq. (3) σ_{S} denotes surface tension, ρ is the liquid metal density, and R is the mean drop radius. This relation shows that the excited oscillations correspond to capillary waves. Physically, small drops are stiffer than bigger drops. By that the oscillation frequency decreases when $\mathrm{Bo}\,(V)$ is increased. The data agree well with the theoretical prediction.

3. Conclusions. We have studied experimentally the static and dynamical behavior of liquid metal drops affected by a high-frequency electromagnetic field. The field is generated by an inductor fed with an alternating current I of a $f = 20$ kHz frequency. We use a self-developed image processing method to analyze the data generated by a high-speed camera system. For $\mathrm{Bo}_{\mathrm{M}} < \mathrm{Bo}_{\mathrm{mc}}$ we observe static symmetric drop squeezing. However, for $\mathrm{Bo}_{\mathrm{M}} > \mathrm{Bo}_{\mathrm{mc}}$ these static states become unstable against capillary waves of a particular mode number m and a particular oscillation frequency f_{C}. We observe that $\mathrm{Bo}_{\mathrm{mc}}$ as well as m and f_{C} depend strongly on the Bond number. Typically, f_{C} are decreased while m is increased when *Bo* becomes bigger. A future task will be to suppress such instabilities using a tailored static magnetic field.

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