A PARADIGMATIC MODEL OF EARTH'S MAGNETIC FIELD REVERSALS

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Introduction. There is an ample paleomagnetic evidence that the Earth's magnetic field has reversed its polarity many times. The last reversal occurred approximately 780000 years ago.The mean rate of reversals varies from nearly zero during the Permian and Cretaceous superchrons to approximately 5 per Myr in the present. Some observations suggest that the decay of the dipole of one polarity might be much slower than the following recreation of the dipole with opposite polarity [1]. Observational data also indicate a possible correlation of the dipole moment with the persistence time of the field in one polarity [2]. In a recent paper, a bimodal distribution of the dipole moment has been hypothesized [3]. Although some of the above mentioned features are controversially discussed in the literature, it is worthwhile to ask if and how they could be modeled by theory. With view on the successful dynamo experiments in Riga and Karlsruhe [4], one could extend this question and ask what would be the most essential ingredient for a dynamo experiment to exhibit irregular reversals in a similar way as the Earth's dynamo does.

Thus motivated, we focus in this paper on a very generic mechanism of field reversals and study it in detail by means of an extremely simple dynamo model. It is well known that the non-self adjoint dynamo operator can provide transitions between non-oscillatory and oscillatory eigenmodes. Typically, this transition occurs at the so-called "exceptional points" [5] of the spectrum, where the eigenvalues and the eigenfunctions of two non-oscillatory modes coalesce and continue as a pair of oscillatory modes with complex conjugate eigenvalues. It is the goal of the present paper to show that even within a very simple mean-field dynamo model the main characteristics of reversals can be attributed to the magnetic field dynamics in the vicinity of such exceptional points.

1. The model. We consider a simple mean-field dynamo model of the α^2 type with a spherically symmetric, isotropic helical turbulence parameter α [7]. The induction equation for the magnetic field $\mathbf{B}$ reads $\dot{\mathbf{B}} = \nabla\times(\alpha\mathbf{B})+(\mu_0\sigma)^{-1}\Delta\mathbf{B}$, with a magnetic permeability μ_0 and an electric conductivity σ. Note that the time scale $\mu_0\sigma R^2$ for theEarth is ~ 200 Kyr, resulting in a free decay time of 20 Kyr for the dipole field. As usual, we decompose $\mathbf{B}$ into a poloidal and a toroidal part, $\mathbf{B} = -\nabla\times(\mathbf{r}\times\nabla S) - \mathbf{r}\times\nabla T$ and expand the defining scalars S and T in spherical harmonics of degree l and order m with the expansion coefficients $s_{l,m}(r,t)$ and $t_{l,m}(r,t)$. For the envisioned spherically symmetric and isotropic α^2 dynamo problem, the induction equation decouples for each degree l and order m into the following pair of equations:

$$\frac{\partial s_l}{\partial t} = \frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(r s_l) - \frac{l(l+1)}{r^2}s_l + \alpha(r,t)t_l \, , \quad (1)$$

$$\frac{\partial t_l}{\partial t} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\mathrm{d}}{\mathrm{d}r}(r t_l) - \alpha(r,t)\frac{\mathrm{d}}{\mathrm{d}r}(r s_l)\right) - \frac{l(l+1)}{r^2}[t_l - \alpha(r,t)s_l] \, . \quad (2)$$

Since these equations are independent of the order m, we have skipped m in the index of s and t. The boundary conditions are $\partial s_l/\partial r|_{r=1} + (l+1)s_l(1) = t_l(1) = 0$. In the following we consider only the dipole field with $l = 1$. For our purpose, we choose a particular radial profile $\alpha(r)$ (with a sign change along the radius) that had been shown to exhibit the oscillatory behaviour [6]. This kinematic $\alpha(r)$ profile is assumed to be quenched with the magnetic field energy which can be expressed in terms of $s_1(r,t)$ and $t_1(r,t)$. In additionto that, we introduce some noise, by which the α-profile is perturbed. Taken all together, $\alpha(r,t)$ can be written as

$$\alpha(r,t) = C \frac{-21.5 + 426.4\, r^2 - 806.7\, r^3 + 392.3\, r^4}{1 + E\left[\frac{2s_1^2(r,t)}{r^2} + \frac{1}{r^2}\left(\frac{\partial(r s_1(r,t))}{\partial r}\right)^2 + t_1^2(r,t)\right]} + \\ +\xi_1(t) + \xi_2(t)\, r^2 + \xi_3(t)\, r^3 + \xi_4(t)\, r^4\,, \tag{3}$$

where the noise correlation is given by

$$\langle \xi_i(t)\xi_j(t+t_1)\rangle = D^2(1 - |t_1|/\tau)\Theta(1 - |t_1|/\tau)\delta_{ij}.$$

C is a normalized dynamo number measuring the over-criticality, D is the noise amplitude, and E is a constant measuring the mean inverse magnetic field energy.

2. Results. First, we consider the case without noise. Fig. 1 shows the magnetic field evolution according to equation system (1)–(3) for $D = 0$ and different dynamo numbers C. The nearly harmonic oscillations for $C = 1.1$ become more and more "shark-fin"-shaped for increasing C, with a pronounced asymmetry between the slow field decay and the fast field recreation during the reversal. At the critical point $C = 1.27893$ a transition to a steady dynamo occurs.

In order to understand this behaviour, we illustrate in Fig. 2 the evolution of the magnetic field ($s(r)$ in Fig. 2*a*, $t(r)$ in Fig. 2*b*, together with the quenched profile $\alpha(r)$ (Fig. 2*c*). In these plots, K always denotes the kinematic dynamo with an unquenched $\alpha(r)$ profile. Numbers 1–8 refer to the snapshots during the reversal that were specified in Fig. 1. From Figs. 2*a* and 2*b* we see how the magnetic field is reversing between snapshots 1 and 8. Fig. 2*c* shows that the profile $\alpha(r)$ changes only slightly during the reversal and that the quenching is concentrated on the inner part of the sphere. At instant 5 it is very close to the kinematic (K) profile, while at instants 1 and 8 it feels the strongest quenching. Fig. 2*d* deserves more explanation. It shows the instantaneous growth rate which would result from the individual $\alpha(r)$ profiles at snapshots 1–8, and from the unquenched

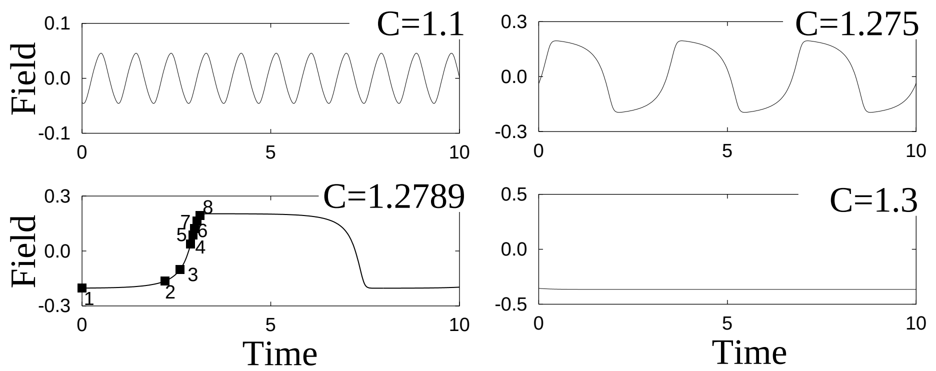

Fig. 1. Magnetic field evolution for $D = 0$ and various values of C. "Field" means $s(r = 0.95)$.

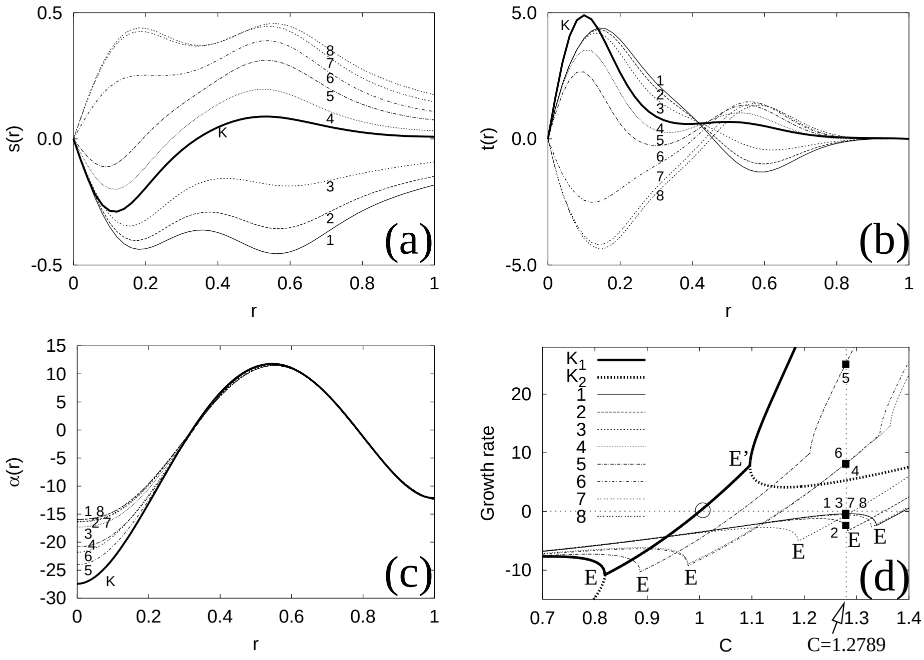

Fig. 2. Explanation of the field dynamics for $C = 1.2789$.

(kinematic) $\alpha(r)$ profile K. In addition to the growth rates at the actual dynamo number $C = 1.2789$, we have plotted the growth rate curves in the interval between $0.7 < C < 1.4$. Only for the kinematic case, we have plotted the branch of the second eigenvalue (K_2) in addition to the first eigenvalue (K_1), both as thick lines. At the exceptional point (the leftmost E) both branches coalesce and continue as a pair of complex conjugate eigenvalues until a second exceptional point E', where they split off again and continue as two separate real eigenvalues. For all other curves, only the exceptional point is indicated by E, whereas the branch of the second eigenvalue has been omitted. In this framework, a reversal can be described as follows: at instant 1, the growth rate "sits" close to the maximum of the non-oscillatory branch, which is slightly below zero. The resulting slow field decay accelerates itself, because the system moves down (snapshot 2) from the maximum of the real branch to the exceptional point. Then the system enters the oscillatory branch (3, 4, 6), with a short intermezzo in the upper non-oscillatory branch (5), which is, however, not essential for the reversal mechanism. Finally, the system moves back again (7, 8) but with the opposite polarity. The critical point $C = 1.27893$ is characterized by the fact that the maximum of the non-oscillatory branch crosses the zero growth rate line. Beyond this point, the field is growing rather than decaying, leading to a stable fixed point somewhere to the right of the maximum of the non-oscillatory branch, and hence to a non-oscillatory dynamo (cf. the case $C = 1.3$ in Fig. 1).

The role of the noise is simply to weaken the sharpness of the critical C. Even above the critical value of C, the noise can trigger a transition to the right of the maximum from where the described reversal process can start (Fig. 3).

3. Conclusion. Here, and in more detail in [8], we have shown that a simple α^2 dynamo model exhibits a number of features, which are typical for the Earth's magnetic field reversals, namely, an asymmetric shape of the reversals, a bimodal field distribution, and correlation of strong fields with long persistence

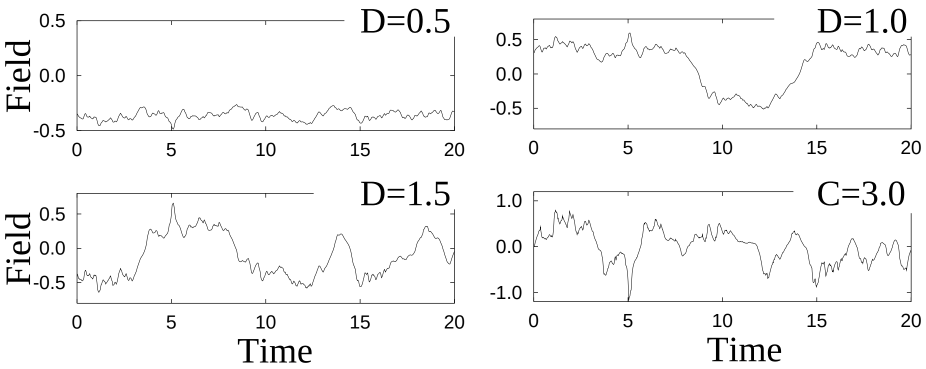

Fig. 3. Time series for $C = 1.3$ and varying intensities of the noise D.

times. We do not claim that our model is an appropriate model of theEarth, although the sign change of $\alpha(r)$ along the radius is not unrealistic [9]. However, it seems worthwhile to try to identify similar reversal scenarios in more complicated models. The gained insight into the important role of spectral exceptional points for the field reversal mechanism might help us to construct dynamo experiments that show reversals, too.

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