COMPUTATIONAL STUDY OF FLOW AND MAGNETIC FIELD INTERACTIONS IN RIGA-DYNAMO

S. Kenjereš[1], *S. Renaudier*[1], *K. Hanjalić*[1], *F. Stefani*[2]

[1] *Department of Multi Scale Physics, Delft University of Technology Lorentzweg 1, 2628 CJ Delft, The Netherlands (kenjeres@ws.tn.tudelft.nl)*
[2] *Forschungszentrum Rossendorf, P.O. Box 510119, D-01314 Dresden, Germany*

1. Introduction. Experimental set-ups for magnetohydrodynamic dynamo involve usually complex configurations of fluid flow and magnetic field, designed to achieve the critical magnetic Reynolds number (Rm) sufficient for magnetic self-excitation, but with minimum energy input and at scales manageable in a laboratory. Computer simulations that could be used to optimize the design and to complement the experiments have, however, been limited to much simplified geometries and flow conditions, which inevitably limit their applicability. In this paper, we report on progress in the coupled *finite difference* (Maxwell solver)/*finite volume* (Navier–Stokes solver) computer simulations of the full-scale real Riga dynamo experiment, Gailitis *et al.* [1]–[3].

2. Results and discussion. Fluid flow in the Riga-dynamo experiment exhibits some features that are very challenging for numerical simulations. Because of very high Re number ($\mathrm{Re} \approx 5 \times 10^6$) needed for ensuring the critical Rm ($15 \leq \mathrm{Rm} \leq 20$), the only option is to apply the Reynolds-averaged Navier–Stokes (RANS) approach. However, accurate predicting of the dynamics of a strong swirl of sodium in the inner annular passage (another prerequisite for achieving critical conditions) is a major challenge for a RANS turbulence model, and so is the flow in the 180^0 loop bend and the subsequent back-flow in the annular passage. We performed first the fluid flow and turbulence simulations for the 1 : 2 scale-down setup with water as working fluid for which detailed experimental database exist. The fluid flow equation set is solved using a finite-volume Navier-Stokes solver for three-dimensional multi-block-structured non-orthogonal domains and high-quality numerical meshes containing up to 2×10^6 cells. Two different turbulence models are considered: a conventional k-ε and a second-moment closure $\overline{u_i u_j}-\varepsilon$ (Reynolds stress) model, Speziale *et al.* [9]. Good agreement has been obtained with the available experimental results, especially with the second-moment closure.

Next we performed full-scale simulations of sodium flow subjected to an approximate magnetic field. The numerical mesh is now extended to cover the walls as well as the outer ring with the sodium at rest. The fluid flow and turbulence simulations are performed first without the Lorentz force. The obtained convergent solutions are exported then to the finite-difference kinematic simulation solver, where the magnetic fields and corresponding Lorentz force are calculated by using the frozen velocity fields. After obtaining converged solutions, electro-magnetic fields are exported and taken into account for the fluid flow and turbulence simulations. In order to preserve the divergency-free magnetic field, an specially developed multi-dimensional interpolation procedure is used for projections of the kinematic-solver results (magnetic field and Lorentz force components) on the underlying finite-volume collocated grid. In this two-step iterative approach, the Lorentz force effects are included in both momentum and turbulence

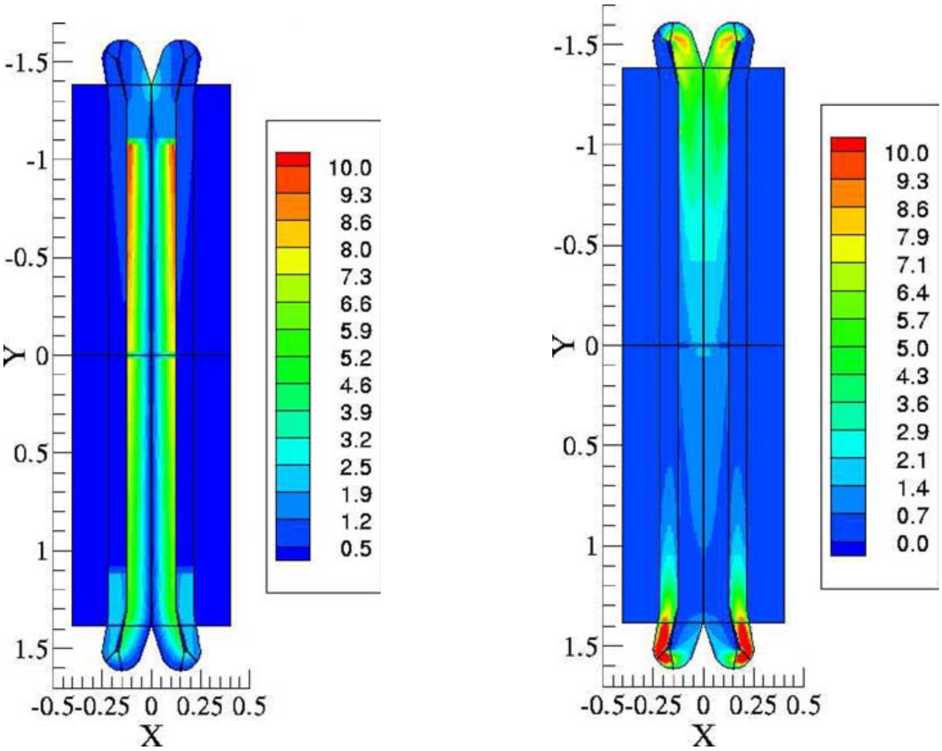

Fig. 1. Distributions of the tangential (left) velocity component and turbulence kinetic energy (right) in the central vertical plane ($z = 0$) of the Riga-Dynamo set-up, Re $= 5.7 \times 10^6$. Results obtained from a finite volume Navier–Stokes solver with two-equations (k-ε) turbulence model, where the Lorentz force effects are included in both momentum and turbulence equations.

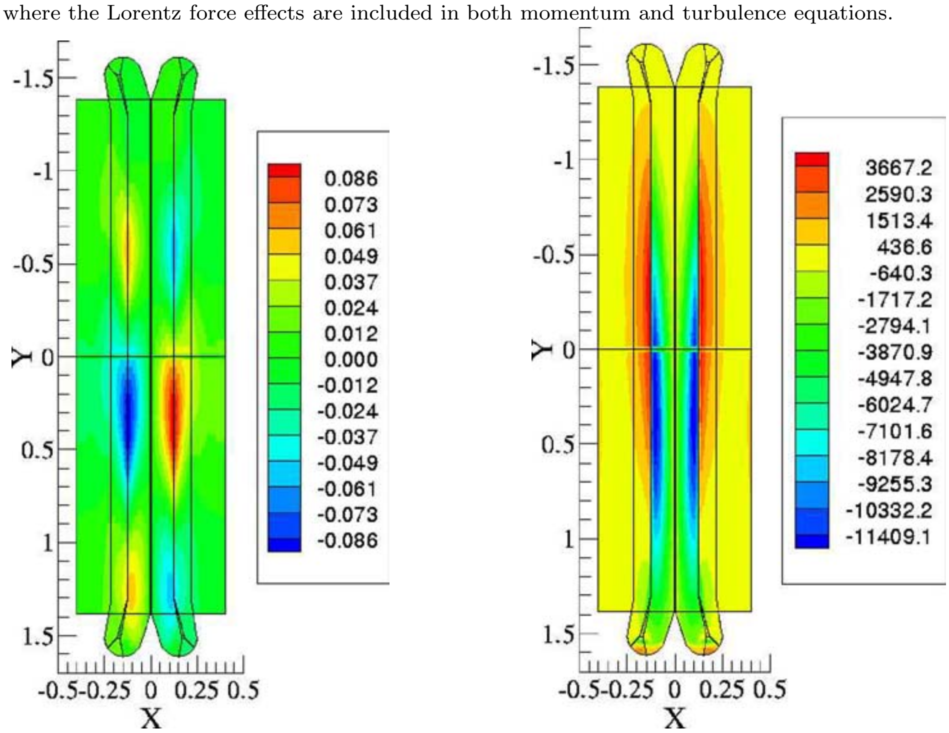

Fig. 2. Distributions of the axial (left) magnetic field component and and tangential (right) Lorentz force in the central vertical plane ($z = 0$) of the Riga-Dynamo set-up, Rm $= 20$. Results interpolated from the 2D finite-difference Maxwell solver.

variable equations. This direct interaction between turbulence quantities and imposed Lorentz force is included via additional terms in transport equations of k, $\overline{u_i u_j}$ and ε, Hanjalić and Kenjereš [5], [6], Kenjereš and Hanjalić [7], Kenjereš *et al.* [8]. It is important to mention that the implementation of the Lorentz force effects into both momentum and turbulence variable equations is the prerequisite for physically correct form of describing turbulent flows subjected to external electro-magnetic effects. As an illustration of the performed numerical studies, distributions of the tangential velocity component and turbulence kinetic energy are shown in Fig. 1 and of the axial magnetic field and tangential Lorentz force in Fig. 2 - both in the central vertical plane. The decay of the tangential velocity component can be nicely observed as well as the regions with very intensive turbulence kinetic energy associated with the strong curvatures of the streamlines. The proper predictions of this decay rate of the tangential velocity are of crucial importance to properly capture the self-excitement regime.

An overview of numerical results, addressing the characteristic growth rates (p) as a function of the rotation rates (Ω)- compared with the measured values in both kinematic and saturation regimes - is shown in Fig. 3. The kinematic two-dimensional finite-difference simulations using the assumed velocity fields (approximated from the 1:2 scale measurements) show good agreement with measurements in the kinematic regime. The saturation regime has been treated by two different models. The first one is a simplified one-dimensional model ("1D model") for the braking of the tangential velocity along the vertical axis, as it was described in Gailitis *et al.* [4]. This model shows significant overpredictions in growth rates compared to the experimental values. Predictions with the velocity fields obtained with the finite-volume Navier–Stokes solver show an improvement in comparison with the previous results, but the problem of the growth-rate overpredictions in the transitional $2000 \leq \Omega \leq 2500$ range is still visible.

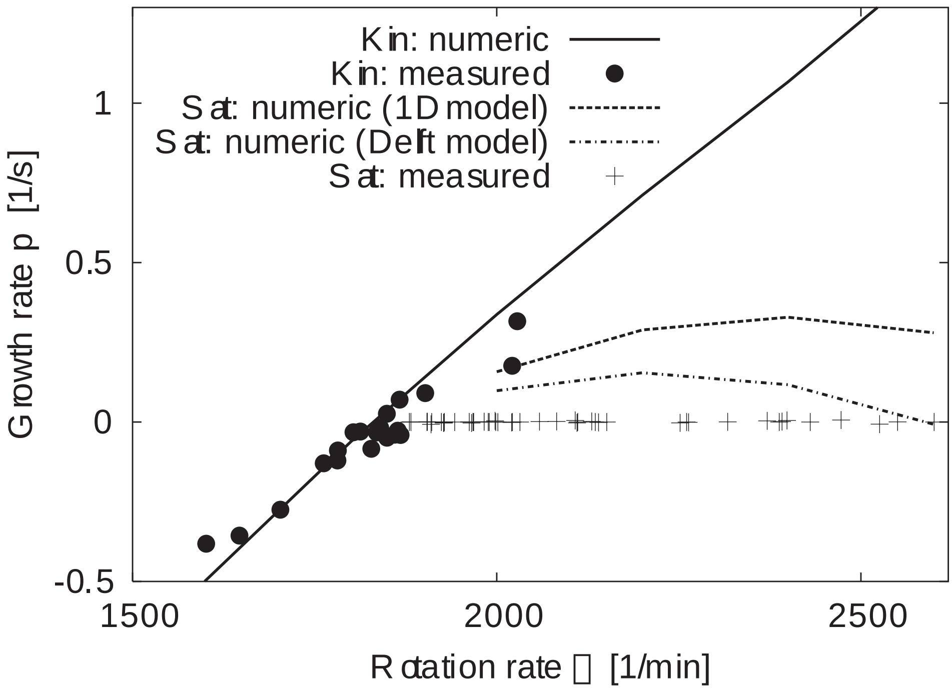

Fig. 3. Growth rate dependence as a function of the rotation rate in kinematic and saturation regimes – comparison between experiments and numerical simulations.

3. Conclusions. The coupled kinematic finite-difference and hydrodynamic solver simulations of the full-scale real Riga dynamo experiment are performed. Because of the very high value of Re number ($\mathrm{Re} \approx 5 \times 10^6$) the RANS approach is used for fluid flow and turbulence predictions. Such obtained velocity fields are then exported to a 2D finite-difference kinematic solver in order to obtain distributions of the electromagnetic parameters (magnetic field and Lorentz force components). These parameters are then exported back to the fluid solver in order to properly account for the back-reaction effects on fluid flow and turbulence. Both direct (through momentum equations) and indirect (through additional 'magnetic' terms in the transport equations for turbulence parameters) effects in fluid flow/turbulence/electromagnetic interactions are taken into account. Significant changes in both velocity components and in turbulence parameters are observed compared to the situation without imposed magnetic field. The coupling between solvers is then finally closed by exporting the velocity components back to the kinematic solver. It is observed that the significant improvements in predictions of the growth rates are observed compared to the uncoupled kinematic solver solutions. There is still room for improvements in the intermediate range of the rotation rates ($2000 \leq \Omega \leq 2500$). Further improvements are expected after development of the fully integrated Navier–Stokes/Maxwell finite-volume solver in the time-dependent mode.

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