GRID-SHELL MODEL OF TURBULENT DISC DYNAMO

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Introduction. The theory of large-scale (mean field) and small-scale (turbulent) dynamos is mainly developed separately. The interaction of these two dynamo mechanisms in real astrophysical systems is a poorly developed topic. Very high values of hydrodynamic and magnetic Reynolds numbers make impossible relevant direct numerical simulations. Some progress in modelling of turbulent MHD-dynamo has been achieved recently using the shell models of turbulence, which also do not describe the turbulent dynamo in all details but allow to reproduce many intrinsic features of the dynamo action in a fully developed turbulence of conductive fluids. It is suggested to combine the mean-field description of large scale dynamo with a shell description of small scale turbulence in a large range of scales. First attempt was done for an α^2-dynamo with the prescribed spatial structure of the large-scale poloidal end and a toroidal magnetic field was considered [1]. We have developed this idea for the case of the $\alpha\Omega$-dynamo in a thin disc taking into account the evolution of the profile of large-scale fields across the disc. The model allows to keep the balance of energy and helicity of the mean and turbulent fields and the flux of these quantities. The suggested approach gives a possibility to study the role of magnetic helicity in the generation process.

1. Model . We consider mean-field magnetic field generation subjected to the joint action of helical turbulence and differential rotation in a thin galactic disc. We use cylindric coordinates r, ϕ, z and measure the length in units of the disc half-thickness, i.e., $0 \le z \le 1$, and we presume that the disc radius is much larger than the disc thickness. Considering a quadrupole magnetic field of axial symmetry, one gets the following set of mean-field equations and boundary conditions

$$\partial_t A = \alpha B + \beta \partial^2_{zz} A, \quad (1)$$

$$\partial_t B = -D \partial_z A + \beta \partial^2_{zz} B \quad (2)$$

$$\partial_z B(0) = 0, \quad B(1) = 0, \qquad \partial_z A(1) = 0, \quad A(0) = 0. \quad (3)$$

Here B is the toroidal magnetic field, A is the toroidal component of the vector-potential responsible for the poloidal magnetic field and D is the so-called dynamo number $D = G\alpha_0\, h^3/\beta_0^2$, here α_0, β_0 are the characteristic values of α and β, $G = r\partial_r\Omega$ is a measure of differential rotation.

To describe the generation of a small-scale magnetic field and its interaction with the turbulent motion, we used a shell-model of MHD-turbulence, introduced by Frick and Sokoloff [2]

$$\mathrm{d}_t U_n = ik_n(\Lambda_n(U,U,a) - \Lambda_n(B,B,a)) - \mathrm{Re}^{-1} k_n^2 U_n + f_n + F_n, \quad (4)$$

$$\mathrm{d}_t B_n = ik_n(\Lambda_n(U,B,b) - \Lambda_n(B,U,b)) - \mathrm{Rm}^{-1} k_n^2 B_n + g_n, \quad (5)$$

$$\Lambda_n(X,Y,c) = c_1 X^*_{n+1} Y^*_{n+2} + c_2 X^*_{n-1} Y^*_{n+1} + c_3 X^*_{n-2} Y^*_{n-1}.$$

Equations are written in the dimensionless form, Re is the Reynolds number, Rm is the magnetic Reynolds number, d_t is the time derivative. The time unit is defined as the turnover time of the vortex on the largest turbulent scale $T = L_0/U_0$. If $a_1 = 1$, $a_2 = -1/4$, $a_3 = -1/8$, $b_1 = b_2 = b_3 = 1/6$, then Eqs. (4)–(5) conserve in the limit $\mathrm{Re}, \mathrm{Rm} \to \infty$ three quadratic quantities, which correspond to three integrals of motion known in magnetohydrodynamics: the total energy $E = E_U + E_B$ (where $E_U = \sum |U_n|^2/2$, $E_B = \sum |B_n|^2/2$), the cross-helicity $H_c = \sum (U_n B_n^* + B_n U_N^*)$ and the magnetic helicity $H_b = \sum (-1)^n |B_n|^2/k_n)$. The term F_n defines the forces, which provide the turbulent flow, the terms f_n and g_n describe the interactions of the mean field and turbulence.

The key point is the conjugation of a mean field and a turbulent fields. We consider the z-depending α in the form

$$\alpha = \sin(\pi z)(\alpha_h + \alpha_m), \tag{6}$$

which includes the hydrodynamic and the magnetic parts of the α-effect. The numerical coefficients are taken from [3]. α_h, α_m are defined by the hydrodynamic helicity χ_h and by the current helicity χ_c, calculated from the shell model as well as the turbulent diffusivity β:

$$\alpha_h = -\frac{1}{3}\sum_n \tau_n \chi_h = -\frac{1}{3}\sum_n \tau_n (-1)^n k_n |U_n|^2 = -\frac{1}{3}\sum_n (-1)^n |U_n|, \tag{7}$$

$$\alpha_m = \frac{1}{3}\sum_n \tau_n \chi_c = \frac{1}{3}\sum_n \tau_n (-1)^n k_n |B_n|^2 = \frac{1}{3}\sum_n (-1)^n |B_n|, \tag{8}$$

$$\beta = \frac{1}{3}\frac{h}{l}\sum_n k_n^{-1}|U_n| + \beta_{\mathrm{Ohm}}, \tag{9}$$

where $\beta_{\mathrm{Ohm}} = 1/\mathrm{Rm}$ is the Ohmic dissipation.

The choice of forces f_n and g_n provides the total energy conservation (however, we ignore the contribution to magnetic energy from differential rotation, i.e., we consider the differential rotation as an unlimited source of energy). It means that these forces have to compensate the energy input, provided by the α-effect. We suggest that the corresponding energy should be removed from the shells, which provide the helicity of dominating sign. Namely, the energy produced by the hydrodynamic and magnetic α-effects are, correspondingly

$$\Delta E_h = 2\int_0^1 \frac{\partial A}{\partial z}\frac{\partial(\alpha_h B)}{\partial z}, \qquad \Delta E_m = 2\int_0^1 \frac{\partial A}{\partial z}\frac{\partial(\alpha_m B)}{\partial z} \tag{10}$$

and the force $f_n = 0$ for even shells and

$$f_n = \frac{U_n \Delta E_h}{E_U} \tag{11}$$

The expressions for g_n are similar (then ΔE_m, E_B and B_n are used).

2. Numerical results. The integration has been done using the fourth-order Runge-Kutta method with adaptive time step, using $\mathrm{Re} = 10^7$, $\mathrm{Rm} = 10^4$. The turbulence was excited by a force with a fixed amplitude and random face in two largest shells. The shell model includes 20 shells ($n = 0, \ldots, 19$). The initial large-scale field was taken in the form

$$A(0,z) = 10^{-3}\sin(\frac{\pi}{2}z), B(0,z) = 0 \tag{12}$$

and the initial values of U_n, B_n were defined as a weak white noise.

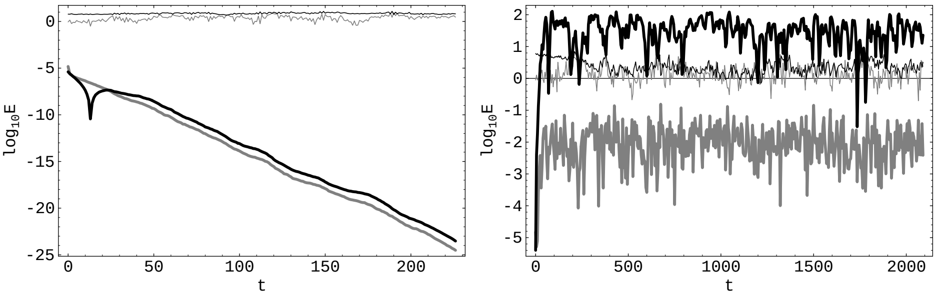

Fig. 1. Large scale magnetic field decay (thick black line – the toroidal field; thick grey line – the poloidal field) in the case $D = -1$ (left) and $D = -100$ (right) when only the hydrodynamic α-effect is taken into account. Thin gray line – small-scale kinetic energy, thin black line – small-scale magnetic energy.

The numerical solutions of the model reproduce many features of the magnetic field dynamo. So, the large-scale field decays for an insufficient large dynamo number (Fig. 1, left). If the dynamo number is large enough, the generation of a large-scale magnetic field in the fully developed turbulence from a random seed field can be seen (we show in Fig. 1, right the case $D = -100$ when only the hydrodynamic α-effect is taken into account). Then the saturation arises with a strong domination of the toroidal field. The dynamo process near the oretical threshold $D \approx -7$ is not stable. For the case $D = -20$, the generation can be stopped temporary (see Fig. 2,(top) due to quenching of the α-effect. After such minima the global field inversion becomes possible (see Fig. 2,(bottom).

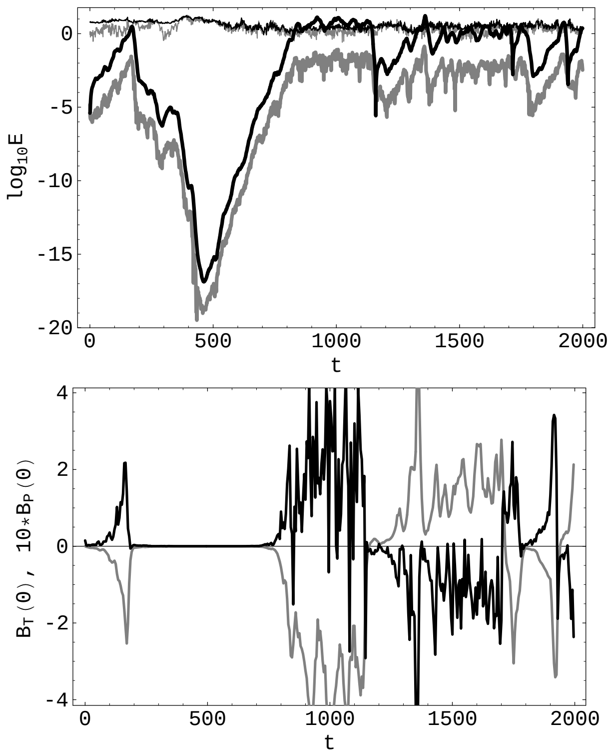

Fig. 2. The energy evolution for $D = -20$ when only the hydrodynamic α-effect is taken into account. Top: thick black line – toroidal field; thick grey line – poloidal field; thin black line – small-scale velocity field; thin gray line – small-scale magnetic field. Bottom: the toroidal and poloidal magnetic field evolution in the middle galactic plane.

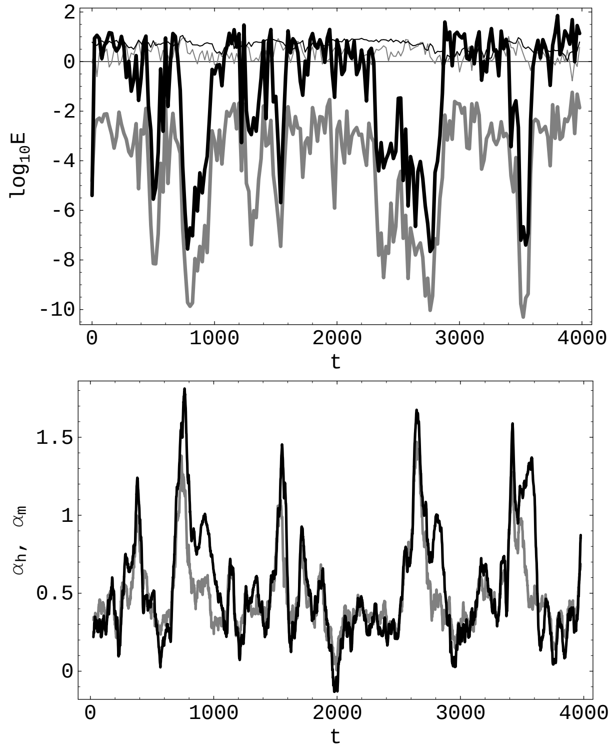

Fig. 3. The energy evolution for $D = -200$ with the full α-effect (6). Top: thick black line – toroidal field; thick grey line – poloidal field; thin black line – small-scale velocity field; thin gray line – small-scale magnetic field. Bottom: the toroidal magnetic field evolution in the middle galactic plane.

If the magnetic α-effect is also included, the character of large-scale dynamo is quite different (Fig. 3, bottom) – frequent decays of the large-scale magnetic field alternate with strong bursts of dynamo activity. This effect of α-quenching is related to the current helicity and has been recently discussed in many papers (see, for example, [4]). One can see in Fig. 3 that the magnetic α-effect suppresses the kinetic one when the large-scale magnetic field decays.

Note that we present here a method to combine a galactic dynamo model for mean-field variables and a shell model for small-scale variables rather than a model of magnetic field generation in a particular galaxy.

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