UNSTEADY ELECTRODYNAMIC PROCESSES ON THE PLANE

G.A. Shaposhnikova, V.A. Buchin

Institute of Mechanics, Moscow University, 1 Michurinskii Pr., 119192 Moscow, Russia

Introduction. In this report the problem of the relaxation of electric charges on weakly conducting plane surfaces is considered. Mylar cloths, polyethylene films, etc., represent the examples of such surface. When these materials undergo certain operations (deployment, spooling, etc.), electric charges appear at certain regions of these surfaces. These charges give rise to sufficiently high electric fields. A time-dependent variation of electric charges and fields on the plane surface is investigated.

1. System of equations. To describe the variation of electric charges and fields, we write a system of equations consisting of the equation for the surface-charge variation and the Coulomb integral law for the electric field due to surface charges (it is assumed that there are no electric charges and electric current beyond the plane under consideration) [1]

$$\frac{\partial q_s}{\partial t} + \mathrm{div}_s \mathbf{j}_\mathrm{s} = 0\,, \qquad \mathbf{E} = \iint\limits_S \frac{4\pi q_s \mathbf{r}}{\varepsilon r^3} \mathrm{d}s. \tag{1}$$

Here, q_s is the surface density of scharge, $\mathbf{j}_s$ is the surface current, div_s is the surface divergence, $\mathbf{E}$ is the electric field vector, S is a charge domain of the surface, $\mathrm{d}s$ is the element of S, ε is the dielectric permeability, and $\mathbf{r}$ is the radius vector directed from the element $\mathrm{d}s$ with the charge q_s to a current point.

At the initial moment, we specify the electric charge distribution $q_s = q_{s0}$ in a bounded region of the surface $S = S_0$.

We consider a symmetric problem when the electric charge in an initial moment takes a strip of width $2L$ and the distribution of the initial charge is symmetric about the central line of this strip. In this case, the problem of charge density variation is one-dimensional, the origin of the coordinate x is the middle line of the strip S_0. The tangential projection E_τ of the electric field is given by

$$E_\tau = \frac{2}{\varepsilon} \int\limits_{-\infty}^{\infty} \frac{q_s(x')}{x - x'} \mathrm{d}x' \tag{2}$$

The variation of charge density $q_s(x)$ is defined by the number of ions that are responsible for the initial charge (b_s^e), their surface mobility as well as by the electrical properties of the surface. The properties listed above define the Ohm's law for the surface current $\mathbf{j}_s$ that appears in the first equation in (1). The numerical solutions for a different form of the Ohm's law for the surface current $\mathbf{j}_s$ are presented in [1]

In this report we consider a simplest case. There is a single type of ions on the surface, namely, the ions that are responsible for the initial electric charge q_{s0}; and there are no other charge carries on the surface. In this model we define the Ohm's law as follows (henceforth, we omit the index τ of the projection E_τ)

$$j_s = b_s^e q_s E. \tag{3}$$

Let us introduce the following dimensionless parameters:

$$q_s^* = \frac{q_s}{q_0}, \qquad E^* = \frac{E}{E_0}, \qquad t^* = \frac{t}{t_0}, \qquad x^* = \frac{x}{L},$$

$$t_0 = \frac{L}{b_s^e E_0}, \qquad E_0 = \frac{2q_0}{\varepsilon}, \qquad q_0 = \frac{1}{2L}\int\limits_{-L}^{L} q_{s0}(x)\mathrm{d}x. \tag{4}$$

The system of equations describing the variation in the charge q_s^* in the dimensionless form is rewritten as follows:

$$\frac{\partial q_s^*}{\partial t*} + \frac{\partial q_s^* E*}{\partial x*} = 0, \qquad E^* = \int\limits_{-\infty}^{\infty} \frac{q_s^*(x')}{x^* - x'}\mathrm{d}x'. \tag{5}$$

Let us introduce new variables τ, ξ, Q, P

$$\begin{aligned} &\tau = \frac{1}{\kappa}\ln(1+\kappa t^*), \quad \xi = \frac{x^*}{(1+\kappa t^*)^\alpha}, \quad \alpha = \text{const} \quad \kappa = \text{const}, \\ &q^* = (1+\kappa t^*)^{\alpha-1} Q(\tau,\xi), \quad E^* = (1+\kappa t^*)^{\alpha-1} P(\tau,\xi). \end{aligned} \tag{6}$$

We can write system (5) using variables (6)

$$\frac{\partial Q}{\partial \tau} - \alpha\kappa\xi\frac{\partial Q}{\partial \xi} + (\alpha-1)\kappa Q + \frac{\partial QP}{\partial \xi} = 0, \qquad P = \int\limits_{-\infty}^{\infty} \frac{Q(\tau,\xi)}{\xi - \xi'}\mathrm{d}\xi'\,. \tag{7}$$

Let us consider the solution of (7) dependent only on $\xi : Q_s = Q_s(\xi)$, $P_s = P_s(\xi)$. The system for this functions has the form

$$-\alpha\kappa\xi\frac{\partial Q_s}{\partial \xi} + (\alpha-1)\kappa Q_s + \frac{\partial Q_s P_s}{\partial \xi} = 0, \tag{8}$$

$$P_s = \int\limits_{-\infty}^{\infty} \frac{Q_s(\xi)}{\xi - \xi'}\mathrm{d}\xi'\,.. \tag{9}$$

The functions $Q_s = Q_s(\xi)$, $P_s = P_s(\xi)$ are the self-similar solutions of equations system (5).

We consider the case then there are no charge carries on the surface except the initial electric charge q_{s0}. In this case the total charge has not changed in the time

$$\int\limits_{-\infty}^{\infty} q(t,x)\mathrm{d}x = Q_f, \qquad Q_f = \text{const}. \tag{10}$$

¿From (10) follows $\alpha = 1/2$. Equation (8) can be written in the form

$$\frac{\partial}{\partial \xi}\left(Q_s\Big(-\frac{1}{2}\kappa\xi + P_s\Big)\right) = 0\,. \tag{11}$$

This equation has the integral

$$Q_s(-\frac{1}{2}\kappa\xi + P_s) = c\,. \tag{12}$$

The constant $c = 0$, as the charge occupies a bounded region. From (12) it follows

$$|\xi| \le \xi_0, \qquad -\frac{1}{2}\kappa\xi + P_s = 0, \tag{13}$$

$$|\xi| > \xi_0, \qquad Q_s = 0\,. \tag{14}$$

Substituting (9) into relation (13) and taking into account (14), we shall write

$$\int\limits_{-\xi_0}^{\xi_0} \frac{Q_s(\xi)}{\xi - \xi'} \mathrm{d}\xi' = \kappa\xi, \qquad \xi_0 = \text{const}. \tag{15}$$

The solution of (15), bounded at the integration limits, has the form [2]

$$Q_s(\xi) = \frac{\kappa}{\pi}(\xi_0^2 - \xi^2)^{1/2}, \qquad |\xi| \leqslant \xi_0. \tag{16}$$

We can calculate the total charge Q_f (10)

$$Q_f = \frac{1}{2}\kappa\xi_0^2. \tag{17}$$

The self-similar constant κ is determined by the total charge Q_f and by the size of the initial charges' region.

REFERENCES

1. G.A. Shaposhnikova. Investigation of unsteady electrodynamic processes on weakly conducting plane surfaces. In *Proceedings of the Steklov Institute of Mathematics*, vol. 223 (1998), pp. 283–287. (in Russ.)

2. F.D. Gakhov. *Boundary Value Problems* (M, Nauka, 1977), p. 640.