SURFACE OF A MAGNETIC FLUID CONTAINING A SPHERICAL BODY IN THE UNIFORM MAGNETIC FIELD

K. Zimmermann[1], *I. Zeidis*[1], *V.A. Naletova*[2,3], *V.A. Turkov*[3], *P.A. Goncharov*[2,3]

[1] *Faculty of Mechanical Engineering, Technische Universitaet Ilmenau, PF 10 05 65, 98684, Germany (klaus.zimmermann@tu-ilmenau.de)*
[2] *Department of Mechanics and Mathematics, M.V. Lomonosov Moscow State University, Leninskiye gory, 119992, Moscow, Russia (naletova@imec.msu.ru)*
[3] *Institute of Mechanics, M.V. Lomonosov Moscow State University, 1 Michurinskiy Pr., 119192, Moscow, Russia*

Introduction. The phenomenon of permanent magnet levitation near the non-deformable boundary of a magnetic fluid (MF) has been first discovered by Rosenzweig [1]. In [2] analytic solutions for a cylindrical magnet inside a cylindrical vessel have been obtained. In [3, 4] the formula for a magnetic force acting on a spherical paramagnetic body and a spherical magnet immersed in the MF inside a spherical vessel in an applied uniform magnetic field and a formula for a force acting on those bodies near a plane that bounds the MF are obtained. In [4] an analogy between the forces that act on a magnet and on a paramagnetic body is proved and it is shown that the paramagnetic body levitation is possible in elipsoidal vessels filled with the MF exposed to uniform magnetic fields. It is clear that the paramagnetic body levitation may be implemented in a layer of the MF, one boundary of which is a free surface. The force that acts on the body depends on the deformation of this free surface and it is important to know the free surface shape. In the present paper we consider the free surface shape of a MF layer, containing a spherical body, in a uniform magnetic field. For some assumptions this problem is solved analytically.

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1. Formulation of the problem. Let us consider a solid spherical body (a is the body radius) made of a magnetizable material with a magnetic permeability μ_b located in the center of the bottom of a cylindrical vessel (R_V is the radius of the vessel) filled with the MF (see Fig. 1). The parameters corresponding to the body, the MF, the material surrounding the MF will be denoted by indices

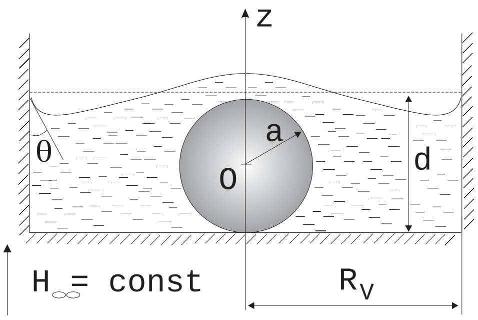


Fig. 1.

b, f, s, respectively. The magnetic permeability of the fluid μ_f, the body material μ_b and the material surrounding the MF μ_s are assumed to be homogeneous, $\mu_f = \text{const} > 1$, $\mu_b = \text{const} \geq 1$, $\mu_s = 1$. An applied uniform magnetic field $\mathbf{H}_\infty$ has the vertical direction. Let us enter cylindrical and cartesian coordinate systems z, ρ, θ and x, y, z with an index point in the center of the sphere. The z-axis is directed upright hill up. In the considered case of cylindrical symmetry the free surface shape can be presented as $z = h(\rho)$ ($\rho = \sqrt{x^2 + y^2}$). The Maxwell's equations and the motion equation are expressed in the form $\text{rot}\mathbf{H} = 0$, $\text{div}\mathbf{B} = 0$, $\mathbf{B} = \mu\mathbf{H}$, $0 = -\boldsymbol{\nabla} p + \rho\mathbf{g}$, $p_{ij} = -pg_{ij} + H_i B_j/4\pi - HBg_{ij}/8\pi$. The conditions on the free surface of the MF and on the body surface $r = \sqrt{x^2 + y^2 + z^2} = a$ are

$$\begin{array}{ll} z = h: & \{p_{ij}n^j\mathbf{e}^i\}_f^s = -2\sigma K\mathbf{n}, \quad \{B_n\}_f^s = 0, \quad \{\mathbf{H}_\tau\}_f^s = 0, \\ r = a: & \{B_n\}_f^b = 0, \quad \{\mathbf{H}_\tau\}_f^b = 0 \end{array} \tag{1}$$

Here $\{A\}_j^i = A^i - A^j$ $i, j = s, f, b$, K is the curvature of the free surface, $2K = (h'' + h'^3/\rho + h'/\rho)/(1 + h'^2)^{3/2}$. The expression for the fluid layer thickness can be written as $h = h_0(1 + \delta(\rho))$. Here $h_0 = d - a$, d is related to the volume of liquid $V = \pi R_V^2 d$ or fluid level near the wall.

We solve the problem using the following approximations: 1) non-induction approximation: $(\mu_f - \mu_s)/\mu_f \ll 1$; 2) long-wave approximation: $h' = h_0\delta' \ll 1$; 3) linear approximation: $\delta \ll 1$.

The non-induction approximation means that the distortion of the applied magnetic field by the MF surface can be neglected and the applied homogeneous field is distorted only by the body. Thus the potential of a magnetic field ϕ equals: $\phi = H_\infty z + AzH_\infty/r^3$, $r = \sqrt{\rho^2 + z^2}$, $A = a^3\alpha = -a^3(\mu_b - \mu_f)/(\mu_b + 2\mu_f)$. The field on the free surface of the MF is easily calculated. Using the equation for the free surface and condition $\delta \ll 1$ the expression for the square of magnetic field on the free surface can be written as:

$$\begin{aligned} & H^2(\rho, z = h) = D_0 + D_1\delta, \ D_0 = \tfrac{9A^2h_0^2H_\infty^2\rho^2}{(\rho^2+h_0^2)^5} + \left(H_\infty + \tfrac{AH_\infty}{(\rho^2+h_0^2)^{3/2}} - \tfrac{3AH_\infty h_0^2}{(\rho^2+h_0^2)^{5/2}}\right)^2, \\ & D_1 = 9\left(-\tfrac{10A^2h_0^4H_\infty^2}{(\rho^2+h_0^2)^6} + \tfrac{2A^2h_0^2H_\infty^2}{(\rho^2+h_0^2)^5}\right)\rho^2 \\ & \quad +2\left(H_\infty + \tfrac{AH_\infty}{(\rho^2+h_0^2)^{3/2}} - \tfrac{3Ah_0^2H_\infty}{(\rho^2+h_0^2)^{5/2}}\right)\left(-\tfrac{9Ah_0^2H_\infty}{(\rho^2+h_0^2)^{5/2}} + \tfrac{15Ah_0^4H_\infty}{(\rho^2+h_0^2)^{7/2}}\right) \end{aligned}$$

In non-induction and long-wave approximations the law of conservation of momentum on the free surface (1) in a projection to the normal is a linear differential equation to define δ:

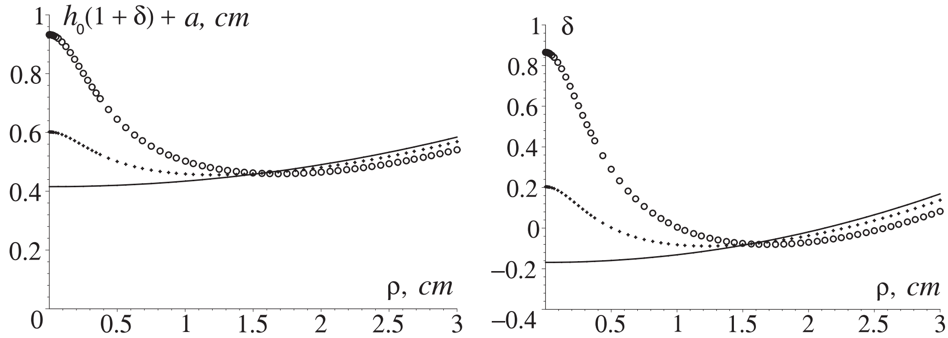
$$\begin{aligned} & \delta'' + \delta'/\rho + f_0\delta = f, \quad f = C_1 - \frac{(\mu_f - \mu_s)D_0}{8\pi\sigma h_0}, \quad C = \text{const}, \\ & f_0 = -\frac{(\rho_f - \rho_s)h_0 g + (\mu_s - \mu_f)D_1/(8\pi)}{\sigma h_0} \end{aligned} \tag{2}$$

Further the gravity is neglected. There is the same effect, if $\rho_1 = \rho_2$. The estimation of values demonstrates that they are the values of the same order. It is possible to consider that $f \gg f_0\delta$ as $\delta \ll 1$. Therefore, $f_0\delta$ from equation (2) is neglected. It allows to obtain analytical solutions and to study the influence of a magnetic field on the MF surface shape. Let us enter the dimensionless parameters:

$$\begin{aligned} & M = -(\mu_f - \mu_s)AH_0^2/8\pi\sigma h_0^2, \quad r_b = a/h_0, \quad \rho^* = \rho/h_0, \quad S = (\rho^{*2} + 1)^{1/2}, \\ & f^* = -M\rho^*(4S + 2\rho^{*2}S - 4r_b^3\alpha - r_b^3\alpha\rho^{*2} - 2\rho^{*4}S)/S^8 + C_2\rho^* \end{aligned}$$

In the dimensionless form equation (2) is written as:

$$\delta''\rho^* + \delta' = f^*(\rho^*, C_2) \tag{3}$$



(*a*) (*b*)

Fig. 2. Line – $H = 0$, points – $H = 300$ Oe, circles – $H = 500$ Oe.

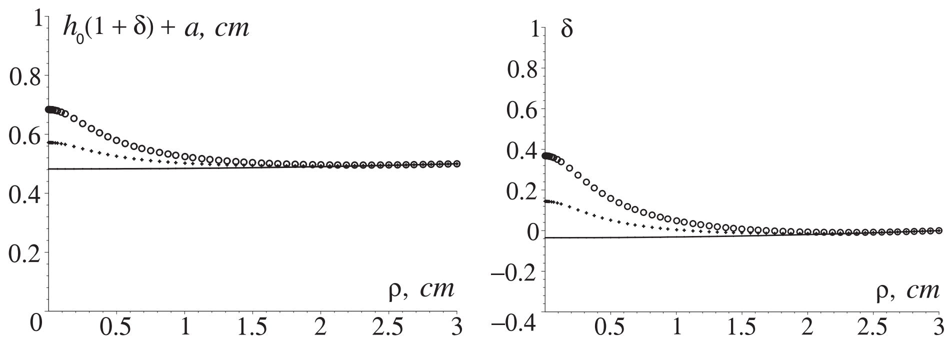
Here $A' = \partial A/\partial\rho^*$. The equation contains an unknown constant C_2 in f^*. If $X = \int f^* \mathrm{d}\rho^*$, $X_0 = X\,|_{\rho^*=0}$, then $\delta = \int((X - X_0)/\rho^*)\mathrm{d}\rho^* + C_3$. The common solution of equation (3), which satisfies the condition of axial symmetry $\delta'(\rho^* = 0) = 0$ is the following:

$$\delta = \frac{2M}{\sqrt{(\rho^{*2}+1)}} + \frac{C_2\rho^{*2}}{4} - \frac{3Mr_b^3\alpha}{8(\rho^{*2}+1)} + \frac{3Mr_b^3\alpha\ln(\rho^{*2}+1)}{8} - \frac{Mr_b^3\alpha}{8(\rho^{*2}+1)^2} + C_3$$

2. Solutions of the problem. We have considered different requirements (boundary conditions on side walls of a vessel and others), which determine the solution of the problem.

Fixed volume of the fluid. Let the volume of MF V be constant and known, Θ be an angle of wetting of the fluid for side walls of a vessel. The parameter d is determined from equation $V = \pi R_V^2 d$. There are conditions: $\partial\delta/\partial\rho^*|_{\rho^*=R_V/h_0} = \cot\Theta$, $\int_0^{R_V/h_0} 2\pi\rho^*\delta\mathrm{d}\rho^* = 0$, from which the coefficients C_2 and C_3 are determined. At the following parameters $h_0 = d - a$, $d = 0.5$ cm, $a = d/3$ cm, $R_V = 3$ cm, $\Theta = \pi/2.1$, $\mu_b \gg \mu_f$, $\mu_f = 1.1$, $\mu_s = 1$, $\sigma = 70$ g/s^2 the dependence of MF thickness $h_0(1+\delta) + a$ and δ on the radius ρ for different magnetic fields are shown in Fig. 2*a*, *b*. Here we calculate δ for an enough large angle $\Theta = \pi/2.1$ (see, Fig. 2*b*). In this case the parameter δ is not small.

Fixed level of a magnetic fluid near the wall of the vessel. After the magnetic field application and the deformation of the MF free surface we add some quantity of the magnetic liquid so that its level near the wall of the vessel is equal to d. In this case the boundary conditions read as: $\delta\,|_{\rho^*=R_V/h_0} = 0$, $\partial\delta/\partial\rho^*|_{\rho^*=R_V/h_0} = \cot\Theta$. The constants C_3 and C_2 are evaluated from these conditions. The constant



(*a*) (*b*)

Fig. 3. Line – $H = 0$, points – $H = 200$ Oe, circles – $H = 300$ Oe.

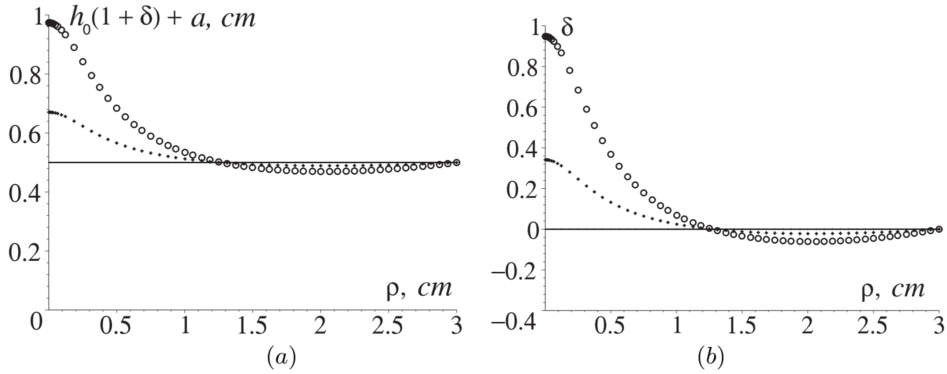


Fig. 4. Line – $H = 0$, points – $H = 300$ Oe, circles – $H = 500$ Oe.

C_2 is the same as for the case of fixed volume of the MF. The solution for the fixed level of the MF near the wall of the vessel is different from the solution for the fixed volume of the MF only in value of the constant C_3. For the enough small $\cot\Theta = \cot(\pi/2.01)$ (and at the following parameters $d = 0.5$ cm, $a = d/3$ cm, $R_V = 3$ cm, $\mu_b \gg \mu_f$, $\mu_f = 1.1$, $\mu_s = 1$, $\sigma = 70$ g/s^2 the dependence of MF thickness and δ on the radius ρ for different magnetic fields (at enough small H, $H \leq 300$ Oe) are obtained (see, Fig. **??**a, b).

In this case and in the case of the fixed volume of the magnetic fluid we can choose the parameters (angle of watering and magnetic field) so that the parameter δ becomes small. Hence, the assumptions made in the given paper are valid.

Magnetic fluid under a thin elastic film. Let us consider the MF under a thin elastic film. In this case the height of the MF near the side wall of the vessel is constant and equals d and the volume of the MF is constant $V =$ const: $\delta\,|_{\rho^*=R_V/h_0}= 0$, $\int_0^{R_V/h_0} 2\pi\rho^*\delta \mathrm{d}\rho^* = 0$. The coefficients C_2 and C_3 are defined from these conditions. Here the coefficient C_2 is different from C_2 in the solutions obtained above. For parameters $d = 0.5$ cm, $a = d/3$ cm, $R_V = 3$ cm, $\mu_b \gg \mu_f$, $\mu_f = 1.1$, $\mu_s = 1$, $\sigma = 70$ g/s^2 the dependence of MF thickness and δ on the radius ρ for different magnetic fields are obtained (see, Fig. 4a, b).

Here the magnitude δ is equal to zero, if the field is absent and at enough small fields the requirement of its smallness is satisfied.

Conclusion. The calculations have shown that for magnetic fields of 100 Oe a noticeable modification of the shape of the surface happens. The obtained analytical solutions can be used for testing of numerical calculations.

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