AMBIGUITY OF THE SHAPE OF A MAGNETIC FLUID DROP IN A MAGNETIC FIELD OF A LINE CONDUCTOR

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Introduction. Phenomena of spontaneous change of the shape of the magnetic fluid (MF) at a slow quasistatic change of system parameters are investigated in papers [1, 2, 3]. Deformation of the MF drop along the wire carrying the current is investigated in [1]. In [2] the jump of the MF to a permanent magnet as it slowly approaches the fluid is studied. In [3] the behavior of the free surface of a boundless MF near the wire carrying the current is studied. There the phenomenon of rapid jump of the ascension height of the fluid has been predicted and observed in experiments. In the present paper the behavior of a drop between solid cylinders in dependence on the current and the volume of the fluid is investigated for all values. Hysteresis of the shape of such drop with the change of the magnetic field is found: the drop breaks up in two drops arising on the inner wire and the external cylinder as the magnetic field increases to critical value and these two drops merge (or not merge) as the magnetic field decreases from the critical value to zero.

1. Formulation of the problem. Consider the problem about an axially symmetric profile $z = z(r)$ of a MF drop located between solid cylinders with the circular cross-section and radii R_c and r_0, $R_c > r_0$ (Fig. 1), r, φ, z are the cylinder
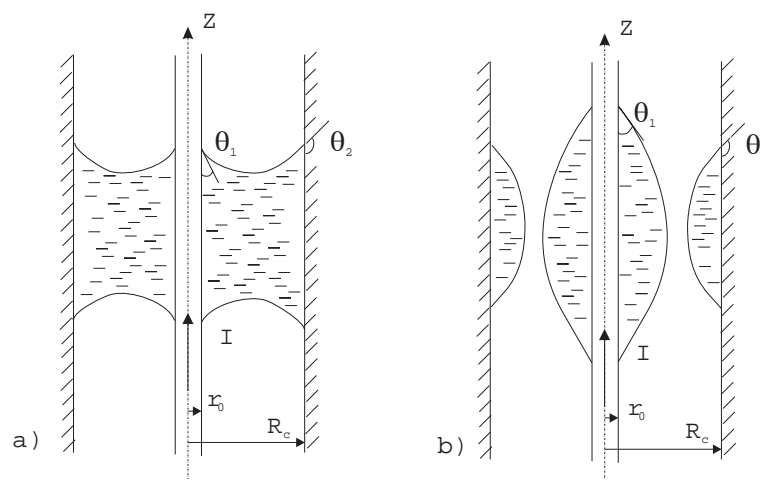

Fig. 1. Magnetic fluid drop (*a*) before break-up (*b*) after break-up.

coordinates. In the bulk of the smaller cylinder the current of intensity I is passing in the wire. In order to neglect gravity, the MF contacts with a non-magnetic liquid of the same density.

The magnetic field is determined by the formula $\mathbf{H} = (0, H, 0)$, $H = 2I/cr$. The equilibrium equation for fluids and the law of momentum conservation for their interface in projection on the outside normal have a form (parameters of the nonmagnetic fluid and the MF are denoted by subscripts (i), $i = 1, 2$ respectively)

$$\frac{\partial}{\partial x_i} p_{ij} = 0, \quad p_{ij} = -\left(p + \frac{H^2}{8\pi}\right)\delta_{ij} + \frac{H_i B_j}{4\pi}, \quad p_{nn}^{(1)} - p_{nn}^{(2)} \pm 2\sigma K = 0 \quad (1)$$

Here K is the mean curvature of the surface, σ is the surface tension of the magnetic fluid. The sign "+" is for the upper interface. In the case of axial symmetry $z = z(r)$, $r = \sqrt{x^2 + y^2}$, the curvature is determined by the relation $2K = (z'' + \frac{1}{r}z'^3 + \frac{1}{r}z')/(1 + z'^2)^{3/2}$. Let us introduce dimensionless parameters

$$\frac{r}{r_0} = r^*, \quad \frac{z}{z_0} = z^*, \quad A = \frac{2\chi I^2}{c^2 \sigma r_0}, \quad B' = \frac{(p_0^{(2)} - p_0^{(1)})2r_0}{\sigma}, \quad R_c^* = R_c/r_0$$

A dimensionless equation for $z^*(r^*)$ can be obtained from (1).

$$-\frac{A}{r^{*2}} + B' = \frac{z^{*''}}{(1 + z^{*'2})^{3/2}} + \frac{1}{r^*}\frac{z^{*'}}{(1 + z^{*'2})^{1/2}} \quad (2)$$

Furtheron, in superscript "*" at related parameters is omitted. The formula for the upper profile of the interface results from (2):

$$z = \int \frac{G}{\sqrt{1 - (G)^2}} dr + D, \quad G = \frac{-A}{r}\ln r + Br + \frac{C}{r} \quad (3)$$

2. Solution and discussion. It is easy to show that the relation $G = -\cos(\theta)$ is valid for the upper interface. Here θ is the angle between the cylinder axis and the tangent to the line $z = z(r)$ (referred to as counter-clockwise). Values of the constants B and C are defined from the boundary conditions at the three phase contact lines $G(r = 1) = -\cos(\theta_1)$, $G(r = R_c) = -\cos(\theta_2)$

$$C = -\cos(\theta_1) - B, \quad B = (A\ln R_c + \cos(\theta_1) + \cos(\theta_2)R_c)/(R_c^2 - 1) \quad (4)$$

For the given wetting angles and the cylinder radii, there exists a critical value of the parameter A, $A_c = \min A$, at which the quantity $|G|$ equals to 1 at some point r, $1 < r < R_c$. For example, at $\cos(\theta_1) = -\cos(\theta_2) = 0.99$ and $R_c = 2$, $A_c = 6.419523443$, $r_c = 1.042605264$, $G(r_c, A_c) = -1$. It means that at $A \geq A_c$ the equilibrium solution does not exist and the drop breaks up. At $A < A_c$ breaking of the initial drop in two parts occurs under a condition that its initial volume V is equaled to V_0 (V_0 is a doubled volume of the liquid between the surface $z = z(r, A)$ and the plane $z = z_{\min}(A)$). The value $z_{\min}(A) = z(r = r_{\min}, A)$, $r_{\min}$ is defined from the condition $z'(r, A) = 0$. The drop between the cylinders exists if $V > V_0$.

The dependence $V_0 = V_0(A)$ has its minimum at $A = A_{\min}$. Furtheron, the variable V_0 grows infinitely as A approaches the critical value A_c. It results from such dependence that if the MF is placed between the cylinders in the absence of the magnetic field (the volume of the drop V is larger than $V_0(A = 0)$), then the splitting of the drop in two parts occurs as $V = V_0(A)$ and the parameter A achieves a value exceeding $A = A_{\min}$. For certainty, let us consider the case $A = 0$,

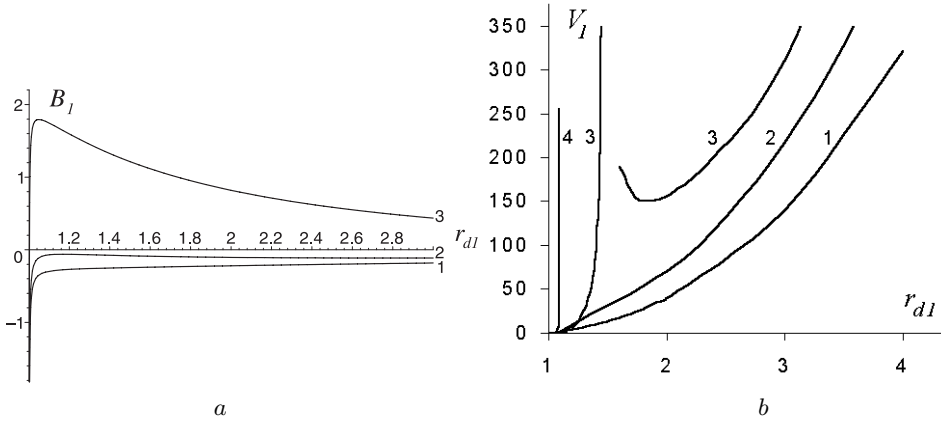*Fig. 2.* Dependences: (*a*) constant B_1 vs. r_{d1}: $1 - A = 0.5$; $2 - A = 1$; $3 - A = 5$; and (*b*) V_1 vs. r_{d1}: $1 - A = 0$; $2 - A = 0.5$; $3 - A = 0.73$; $4 - A = 5$.

$V = 2.383711614$. Calculations (for $\cos(\theta_1) = -\cos(\theta_2) = 0.99$ and $R_c = 2$) show that the drop breaking in two parts for the volumes $V_1 = 1.59$ and $V_2 = 0.79$ occurs at $A = 5$. Subscripts 1 and 2 denote drops on the wire and on the external cylinder, correspondingly. Further on a possibility of merging of these drops with the decreasing A is considered.

Drop on the wire with the current. The equation describing the shape of the drop on the wire with the current has a form

$$z = \int_1^r \frac{G_1}{\sqrt{1 - G_1^2}} dr, \quad G_1 = \frac{-A}{r} \ln r + B_1 r + \frac{-\cos(\theta_1) - B_1}{r} \tag{5}$$

Here z is a function of r – the upper limit of the integral, which is confined by the drop thickness r_{d1}. Consider the case $\cos(\theta_1) = 0.99$. It is shown that the relation between the unknown constant B_1 and the maximal radius of the drop r_{d1} can be yielded from the equation $G_1(r = r_{d1}) = -1$. If there are several roots of this equation, the root larger than 1 and nearest to 1 is chosen. Knowing r_{d1}, we can calculate B_1 and the volume of the drop V_1. Dependence B_1 on r_{d1} at $A = 0$, $A = 2$, $A = 5$ is illustrated in Fig. 2a.

It is shown that for small $A < A_1 = 0.7297722116$ the dependence B_1 on r_{d1} is increasing monotonously and one-two-one correspondent, see curve 1 in Fig. 2a. At $A_1 < A < A_2 = 1.144716816$ this curve has two extrema, $B_1 < 0$ and there exists a region r_{d1}, which cannot be obtained, see curve 2 in Fig. 2a. At $A > A_2$ the quantity B_1 has a positive maximum, see curve 3 in Fig. 2a. The calculations show that one-two-one correspondence is observed only at $A < A_1$ and $A > A_2$. For intermediate values $A_1 < A < A_2$ the three values of the radius r_{d1} correspond to one value of the volume. The dependence of the volume of the drop r_{d1} is presented in Fig. 2b for various A at $\cos(\theta_1) = 0.99$. Multivalence of the solution is shown to be obtained at a sufficiently large volume of the drop $V_1 > V_{1c}$, $V_{1c} = 150$. On the other hand, the volume of the drop on the wire arising due to breaking of the initial bridge at $A = 5$ is small enough, $V_1 = 1.593 < 150$. Hence, the decrease of the magnetic field (the parameter A) to zero results in a slight increase of the drop thickness from $d_1(A = 5) = 0.049$ to $d_1(A = 0) = 0.13$.

Drop on the surface of the external cylinder. An equation describing the shape of the second drop has a form

$$z_2 = \int_1^r \frac{G_2}{\sqrt{1 - G_2^2}} dr, \; G_2 = \frac{-A}{r} \ln r + B_2 r + \frac{C_2}{r}, \; C_2 = A \ln R_c - R_c(B_2 + \cos(\theta_2))$$
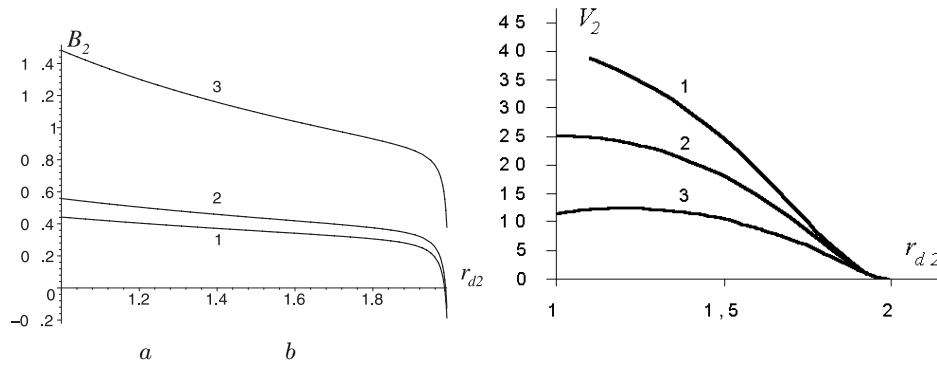
Fig. 3. Dependences: (*a*) B_2 vs. r_{d2}: $1 - A = 0$, $2 - A = 2$; $3 - A = 5$; and (*b*) V_2 vs. r_{d2}: $1 - A = 0$, $2 - A = 2$; $3 - A = 5$.

Here z is a function of r (the upper limit), which is restricted below by the value r_{d2}, $r_{d2} < r < R_c$. Consider the case $\cos(\theta_2) = -0.99$ and $R_c = 2$. Knowing the root of the equation $G_2(r = r_{d2}) = 1$, we can obtain a relation between the unknown constant B_2 and r_{d2} $B_2 = -(-A\ln(r_{d2}) + A\ln(R_c) - \cos(\theta_2)R_c - r_{d2})/(r_{d2}^2 - R_c^2)$. Dependence of B_2 on r_{d2} at $A = 0$, $A = 2$, $A = 5$ (at $\cos(\theta_2) = -0.99$ and $R_c = 2$) is presented in Fig. 3a.

It is found that the dependence B_2 on r_{d2} is decreasing monotonously and one-two-one correspondent for any $A > 0$. The function V_2 in dependence on r_{d2} has a maximum for certain values of the parameter A, see curve 3 ($A = 5$) in Fig. 3b). It is meas that at $A = A_{cr\ 2}(V_2)$ the drop on the external cylinder breaks up in two drops or becomes a drop between two cylinders. For our problem the volume of the drop on the external cylinder is equal to $V_2 = 0.79$. The dependence of the drop thickness d_2, $d_2 = R_c - r_{d2}$, on the parameter A (the magnetic field) is monotonous: as the parameter A decreases from 5 to 0, the drop thickness decreases from 0.06445 to 0.06153. It is seen that switching off the magnetic field does not make the drops merge; $d_1 + d_2 = 0.197 < R_c - 1 = 1$.

Conclusion. It is shown that a sufficiently small MF drop placed between two cylinders breaks up in two drops arising on the inner wire and on the external cylinder, as the magnetic field increases. These drops do not merge at decreasing of the magnetic field to zero. It means that a specific hysteresis of the shape can be observed as the magnetic field increases and decreases. It is found that the dependence of the drop volume on the wire on the drop thickness is many-valued for some parameters of the problem. This results in hysteresis behavior of the drop thickness at the subsequent increasing and decreasing of the magnetic field. It is shown that the drop located on the external cylinder can exist only if the magnetic field does not exceed some critical value, and there is no hysteresis if the field is less than this critical value.

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