DYNAMICS OF ANISOTROPIC FLEXIBLE MAGNETIC FILAMENTS

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Introduction. The flexible magnetic filaments created in different labs [1, 2] recently have attacted a great interest. The theoretical model based on the extension of the Kirchhoff model of the elastic rod by including the magnetic energy term was developed in [3]. On its basis the dynamics of the filament in the rotating field [4], bending by the applied transversal force [5] were considered. By introduction of the anisotropy of the friction coefficient a possibility to create micromachines driven by an AC magnetic field was predicted by numerical simulations [6]. In [6] to obtain the steady selfpropulsion of the filament with a hairpin shape, its magnetic heterogeneity was considered. Here the issues of the anisotropy of the filament, which can also have magnetic origin due to magnetodipolar interactions, and heterogeneity, are considered in more details. Annother motivation of the present work comes from experimental observations of steady "U"-like filament shapes in the rotating field [2] which, as it was illustrated in [4], are unstable. One possibility to explain the formation of such shapes consists in the assumption of magnetic heterogeneity of the filaments due to irregularities in the process of their production. Here we illustrate by numerical simulations that in the case of the magnetic heterogeneity "U"-like shape synchronously rotating with the magnetic field is possible.

1. Model. According to a Kirchoff model of elastic rod extended by including the magnetic term, its energy has the form [3]:

$$E = \frac{1}{2} C \int \frac{1}{R^2}\,\mathrm{d}l - \frac{2\pi^2 a^2 \chi^2 H_0^2}{\mu + 1} \int (\mathbf{ht})^2\,\mathrm{d}l - \int \Lambda \mathrm{d}l \qquad (1)$$

Here R is the radius of curvature of the center line but C through the radius of a cylinder and its Young modulus Y is expressed as follows $C = \frac{\pi}{4} a^4 Y$, $\mu = 1 + 4\pi\chi$, χ is the magnetic susceptibility of the rod, $H_0\mathbf{h}$ is the external magnetic field strength in the direction of the unit vector $\mathbf{h}$. The tangent to the center line $\mathbf{t}$ is given by its components $\mathbf{t} = (\cos\vartheta, \sin\vartheta)$. Local inextensibility of the rod is accounted for by introducing local tension of the rod as a Lagrange multiplier Λ.

Considering the variation of (1) with respect to $\mathbf{r}' = \mathbf{r} + \xi$ from the first variation of the energy functional

$$\delta E = [M\delta\varphi] + [F_t \xi_t] + [F_n \xi_n] - \int K_t \xi_t\,\mathrm{d}l - \int K_n \xi_n \mathrm{d}l, \qquad (2)$$

momentum stresses M, normal and tangential stresses F_n, F_t as well as body force $\mathbf{K}$ are found in [3].

Here [ ] denotes the values at the ends of the rod, ξ_n and ξ_t are the components of Lagrange displacement in the directions of the normal and tangent to the center line correspondingly, but $\delta\varphi = \frac{\partial \xi_n}{\partial l} - \frac{\xi_t}{R}$ is the angle of a tangent angle rotation at the Lagrange displacements $\boldsymbol{\xi}$.

Introducing the characteristic elastic time scale $\tau^{-1} = \frac{C}{L^4\zeta}$, length scale L, where $2L$ is the length of the rod, the set of equations for tangent angle and tension has the following dimensionless form

$$\vartheta_t = -\left(\vartheta_{llll} + \frac{1}{2}\left(\vartheta_l^3\right)_l\right) - \left(\vartheta_l \Lambda\right)_l - \Lambda_l \vartheta_l + \mathrm{Cm}\left(\sin\left(2\vartheta\right)\right)_l - \mathrm{Cm}\,\vartheta_l^2 \sin\left(2\vartheta\right) . \quad (3)$$

$$\vartheta_l^2 \Lambda - \Lambda_{ll} = -\vartheta_l\left(\vartheta_{lll} + \vartheta_l^3\right) + 2\mathrm{Cm}\,\vartheta_l^2 \cos\left(2\vartheta\right) + \mathrm{Cm}\left(\vartheta_l \sin\left(2\vartheta\right)\right)_l . \quad (4)$$

Here $\mathrm{Cm} = \frac{2\pi^2 a^2 \chi^2 H_0^2 L^2}{C\left(\mu+1\right)}$ is the magnetoelastic number characterizing the ratio of the magnetic and elastic forces. Coupled set of equations (3) and (4) describing the dynamics of the rod under the action of the magnetic and elastic forces in the viscous fluid is solved numerically. The boundary conditions corresponding to the free and unclamped ends of the filament are the following:

$$-\vartheta_{ll} + \mathrm{Cm}\sin\left(2\vartheta\right) = 0, \quad (5)$$

$$\vartheta_l = 0 \quad (6)$$

and

$$\Lambda = 0. \quad (7)$$

2. Anisotropic model. In the case when the anisotropy of the friction coefficient is taken into account the components of the velocity can be written as follows

$$\beta\zeta v_n = K_n, \quad (8)$$

$$\zeta v_t = K_t, \quad (9)$$

where ζ is the friction coefficient in the tangential direction and β is the ratio between friction coefficients in normal and tangential directions. In this case model equations (3)–(4) read

$$\begin{aligned}\beta\left(\vartheta_t + \omega\tau\right) = -\left(\vartheta_{llll} + \frac{1}{2}\left(\vartheta_l^3\right)_l\right) - \left(\vartheta_l\Lambda\right)_l - \beta\vartheta_l\Lambda_l + \\ +\mathrm{Cm}\left(\sin\left(2\vartheta\right)\right)_{ll} - \beta\mathrm{Cm}\,\vartheta_l^2\sin\left(2\vartheta\right) ,\end{aligned} \quad (10)$$

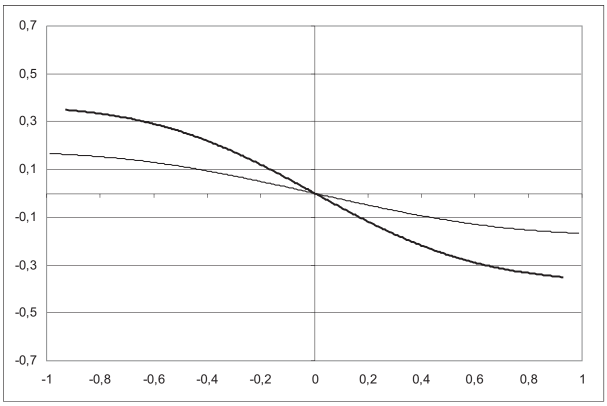

Fig. 1. Configuration of the rod in the rotating magnetic field in the frame of the field. $\mathrm{Cm} = 25$, $\omega\tau = 25$. Direction of magnetic field rotation is anticlockwise. Solid line – $\beta = 1$, bold line – $\beta = 2$.

$$\vartheta_l^2\Lambda - \beta\Lambda_{ll} = -\vartheta_l\left(\vartheta_{lll} + \frac{1}{2}\vartheta_l^3\right) + 2\mathrm{Cm}\,\vartheta_l^2\cos(2\vartheta) + \beta\,\mathrm{Cm}\left(\vartheta_l\sin(2\vartheta)\right)_l\,. \quad (11)$$

The anisotropy coefficient β is between 1.5–2 [7]. In the case when β is equal to one, we have isotropic model (3)–(4).

As seen from Fig. 1, the anisotropy of the friction coefficient increases the phase lag of the magnetic rod from the magnetic field (bold line) in comparison with the isotropic case (solid line). Thus the anisotropy of the friction coefficient of the filament has influence on its steady shapes in the rotating magnetic field.

Due to the anisotropy of the magnetodipolar interactions, the curvature elasticity of the magnetic filaments can be anisotropic. In this case the energy expression is

$$E = \frac{1}{2}C\int\frac{1}{R^2}\,\mathrm{d}l - \frac{1}{2}\gamma C\int\frac{\sin\vartheta}{R^2}\,\mathrm{d}l - \frac{2\pi^2a^2\chi^2H_0^2}{\mu+1}\int(\mathbf{ht})^2\,\mathrm{d}l - \int\Lambda\,\mathrm{d}l\,. \quad (12)$$

The equations for the tangent angle and tension are derived as follows

$$\begin{aligned}\vartheta_l^2\Lambda - \beta\Lambda_{ll} = &-\vartheta_l\big(\vartheta_l(1-\gamma\sin^2\vartheta)\big)_{ll} - \frac{1}{2}\vartheta_l^4\big(1-\gamma\sin^2\vartheta\big)+\\ &+\beta\mathrm{Cm}\big(\vartheta_l\sin(2\vartheta)\big)_l + \mathrm{Cm}\vartheta_l\big(\sin(2\vartheta)\big)_l + \frac{1}{2}\vartheta_l\big(\vartheta_l(1-\gamma\sin^2\vartheta)_l\big)_l \end{aligned} \quad (13)$$

$$\begin{aligned}\beta(\vartheta_t + \omega\tau) = &-\big(\vartheta_l(1-\gamma\sin^2\vartheta)\big)_{lll} - \frac{1}{2}\big(\vartheta_l^3(1-\gamma\sin^2\vartheta)\big)_l - (\Lambda\vartheta_l)_l - \beta\Lambda_l\vartheta_l+\\ &+\mathrm{Cm}\,\big(\sin(2\vartheta)\big)_{ll} - \beta\mathrm{Cm}\,\vartheta_l^2\sin(2\vartheta) + \frac{1}{2}\big(\vartheta_l(1-\gamma\sin^2\vartheta)_l\big)_{ll}\,.\end{aligned} \quad (14)$$

If the coefficient $\gamma = 0$, the set of equations (13)–(14) transforms to the considered above (10)–(11). The same boundary conditions are valid, the case with ($\beta = 1$) is further considered.

In Fig. 2 the shape of the filament with the anisotropy of the curvature elasticity in the rotating field is compared with that of isotropic case. It can be seen that the magnetic anisotropy of the filament diminishes the phase lag from the magnetic field. This seems to be natural since the effective curvature elasticity decreases with the increase of the phase lag.

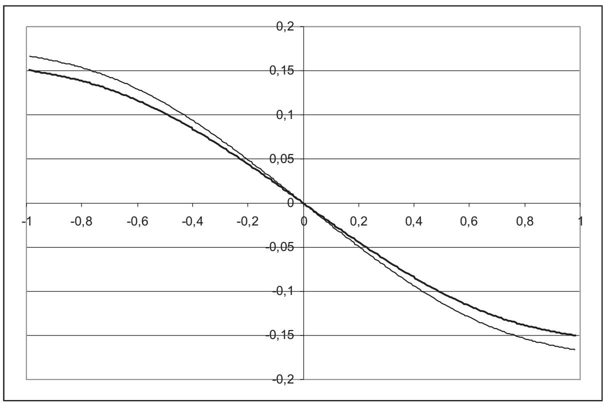

Fig. 2. Configuration of the rod in the rotating magnetic field in the frame of the field. $\mathrm{Cm} = 25$, $\omega\tau = 25$, $\beta = 1$. Direction of magnetic field rotation is anticlockwise. Solid line – $\gamma = 0$, bold line – $\gamma = 1$.

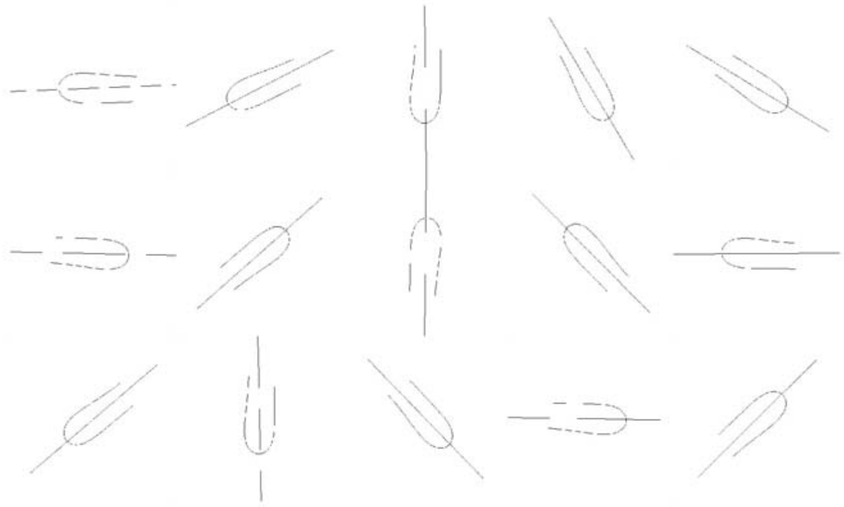

Fig. 3. Configurations of a flexible magnetic chain from the point of view of the laboratory set of coordinates. Straight line shows the magnetic field direction. $\omega\tau = 45$. From top right to left down $t = 0.000889$, 0.01067, 0.03511, 0.04684, 0.05760, 0.06933, 0.08596, 0.10453, 0.12213, 0.13973, 0.15538, 0.17493, 0.19156, 0.20916, 0.22676.

3. Heterogeneus filament. To consider the influence of the magnetic heterogeneity of the filament on its dynamics in the rotating field, a diminished value of the magnetoelastic number around the center of the filament has been taken. As it was shown earlier [4], the “U”-shape is not stable in the case of rotating magnetic field. It sooner or later relaxes to the shape close to the shapes shown in Fig. 1. In Fig. 3 the steady "U" like shape synchronously rotating with the field is shown. It is found for the following set of parameters $\omega\tau = 45$, Cm $= 25$; $l \in [-1; -2/25] \cup [2/15; 1]$ and Cm $= 25$*0.7; $l \in [-2/15; 2/15]$. This result gives the possibility to explain the observation of "U" - like shapes of the filaments in experiment [2] by their magnetic heterogenity presumably arising during the process of the production of the filaments.

4. Conclusions. Different models of magnetic filaments studied here show that the anisotropy induced by long-range hydrodynamic and magnetic interactions has influence on the steady shapes of filaments in the rotating field. Observation in experiments of the peculiar "U" - like shapes can be associated with magnetic heterogenity of the filaments arising at their production.

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