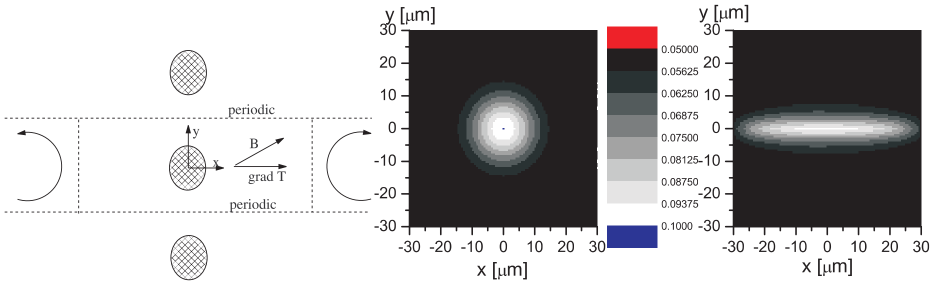
MICROCONVECTION AND MASS TRANSFER NEAR BODIES IN NON-UNIFORMLY MAGNETIZED FERROFLUID

V. Frishfelds, E. Blums

Institute of Physics, University of Latvia, 32 Miera, LV-2169 Salaspils, Latvia (frishfelds@latnet.lv)

Introduction. Experimental observations of magnetic fluid behaviour around a grid subjected to a magnetic field shows that the presence of a temperature gradient creates a notable separation of the fluid between both sides of the grid with different temperatures [1]. Some separation of the ferrofluid occurs also without a magnetic field. Our aim here is to discuss in more detail the ferrofluid convection and subsequent separation under the action of magnetic field force resulting from the magnetic Soret effect. Understanding of the directed mass transfer in a non-uniformly magnetized ferrofluid is still insufficient, and different opinions about the role of various mechanisms are presented. Therefore, a detailed description of heat and mass transfer processes are required [2]. The heat and mass transfer processes are analysed numericaly by a two-dimensional macroscopic model in order to estimate the role of convection around micron size bodies (microconvection). Particularly, a grid consisting of large number of equidistantly spaced non-magnetic or magnetic cylinders is considered. The example of the system is shown in Fig. 1a. Non-uniform magnetization of the ferrofluid surrounding each cylinder creates an oriented microconvection, which is the main reason for appearance of the magnetic Soret effect. Therefore, distributions of temperature, magnetic field, concentration, and velocity have to be obtained for a correct description of the microconvection.

1. Equations. Despite the fact that the convective heat transport is negligible, a non-uniform temperature gradient exists as the cylinder can have a higher heat conductivity λ. Because the convective heat transfer is negligible in comparison with the conductive one, the stationary temperature distribution is only required $\nabla(\lambda\nabla T) = 0$. Magnetic field calculation is performed using a scalar magnetic potential. The stationary magnetic scalar potential obeys $\nabla(\mu\nabla\Psi) = 0$ and the magnetic intensity is given by $\mathbf{H} = \nabla\Psi$. According to the Langevin equation, a relative magnetic permeability μ depends both on the temperature



a *b*

Fig. 1. (*a*) Schematic shape of the grid in the system surrounded by a ferrofluid. (*b*) Development of the inhomogeneous concentration area in a magnetic field (B=0.1 T) oriented horizontally and without bodies.

and concentration. The magnetic susceptibility is calculated at an average temperature of the system by the Langevin equation and adding an experimentally observed dependence of susceptibility on the temperature. Magnetic force density is approximated either by the Kelvin formula $\mathbf{f} = \mu_0 \mathbf{M}\nabla\mathbf{H}$ or an extension to the Helmholtz one [3]. The Helmoltz force accounts for the nonlinear dependence on the concentration of ferrofluid and givesa higher value of the resulting Soret coefficient. The gravitational force is usually smaller than the magnetic one for particles with a diameter smaller than 10 nm. The magnetic force acting on ferrofluid particles drives the fluid flow and causes a redistribution of ferrofluid concentration. An incompressible fluid flow is calculated from the Navier-Stokes equations, which are transformed to the shape of the velocity potential ψ and vorticity ζ in a two-dimensional case

$$\partial_t \zeta + (\mathbf{v}\nabla)\zeta = \frac{1}{\rho}\left(-\nabla \times (\nabla \times (\eta\zeta)) + \nabla \times \mathbf{f}\right), \quad \zeta = \nabla \times \mathbf{v}, \quad \mathbf{v} = \nabla \times \psi. \quad (1)$$

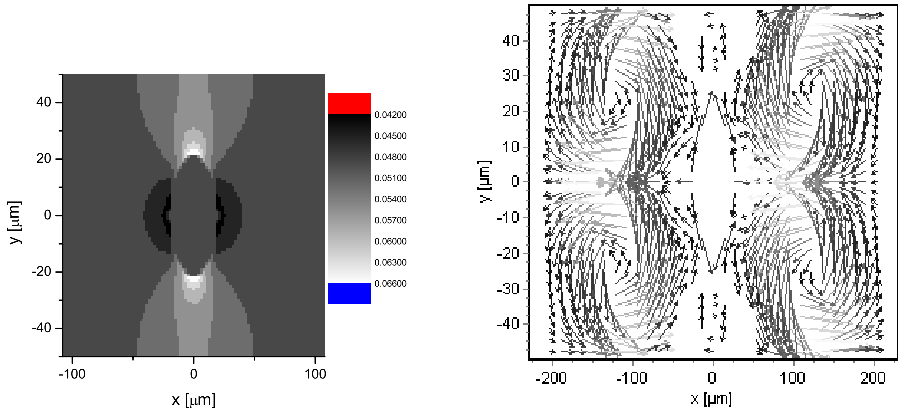
The dynamic viscosity η is a function of the temperature but the fluid density is assumed to be constant. The role of density variations will be considered in the future. Typically, the resulting Reynolds numbers for the given system are very small, resulting in a laminar behavior of the fluid flow. Because the magnetic force acts on nanoscale ferroparticles of the fluid, the concentration of those is no more constant. The distribution of the ferrofluid concentration c (volume fraction of ferroparticles) is described by the mass conservation equation with drift, diffusion, and convection terms. If the ferroparticles are spherical and of equal radius r_p, the ferroparticle flux in Stokes formulation is

$$\mathbf{j} = \mathbf{v}c - D\nabla c + \frac{2r_p^2}{9\eta}\mathbf{f}, \quad D = \frac{kT}{6\pi r_p \eta}. \quad (2)$$

The distribution of the concentration can be then calculated from the mass conservation law $\partial_t c + \nabla \mathbf{j} = 0$.

2. Numerics. Analytical two-dimensional solutions of the temperature, magnetic field, magnetic force and stationary flow field are possible for simple geometries at low Reynolds numbers especially at a uniform distribution of the concentration. Estimations of the flow field have been obtained for a cylinder placed in an infinite ferrofluid, which has the permeability linearly varying with the temperature. These estimations have been generalized for a series of equidistantly placed cylinders. The analytical solution of velocity shows that a six-vortex regime is characteristic for the flow around the cylinder. The fluid flow does not decay far from one cylinder, i.e., the flow of ferrofluid is present over the whole area with the varying magnetic permeability.

The Soret coefficient and the osmotic pressure difference can be obtained only taking into account the varying concentration of ferrofluid. Therefore, a transient two-dimensional numerical model has been constructed to find all distributions in the cell. Numerical description enables to consider not only cylinders but a large variety of bodies. It is possible to switch from description in the plane coordinate system to axial symmetric coordinates that enables to calculate microconvection, e.g., around the sphere. The temperature distribution is calculated only once as it has no time dependence. On the other hand, the distribution of magnetic field is recalculated after each time step as the magnetic permeability depends on the concentration of ferroparticles. Separate or simultaneous calculation of the flow and concentration fields is performed afterwards. If the magnetic field is highly



a *b*

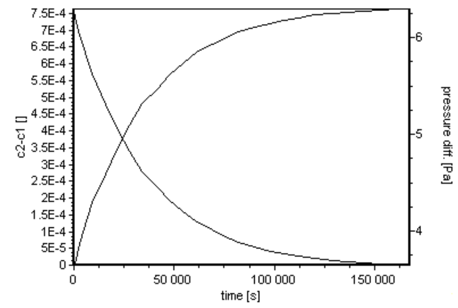
Fig. 2. (*a*) Stationary flow around an elliptic grid element. (*b*) Stationary distribution of the concentration. Magnetic field $B = 0.0375\,\mathrm{T}$ is oriented along the temperature gradient. Average ferrofluid concentration by volume fraction is 0.05.

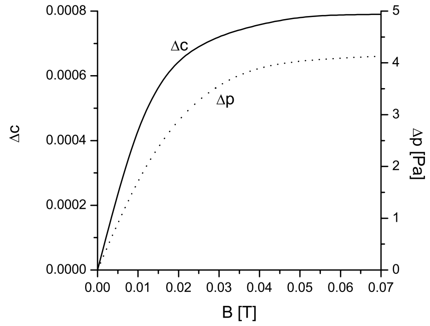
coupled with the concentration field as it is for high concentrations and nonmagnetic bodies, the magnetic field, flow field, and concentration can be calculated simulateneosly to allow larger time steps. The model is accompanied by automatic generation of mesh and time step to reduce the necessary intervention. The finite volume method is used to obey the mass conservation laws strictly.

3. Boundary conditions. Particular attention should be paid to the boundary conditions of the model as the perturbations decay very slowly far from the bodies. Because the bodies in Fig. 1a are placed equidistantly, only the periodic part of the system has to be calculated. The periodic boundaries do not require any special conditions except the magnetic and velocity potentials. The difference of the velocity potential between the both sides is obtained either by setting the total flow of fluid through the system to zero or setting the pressure difference between far away boundaries. It is assumed that the tangential velocity component and the tangential pressure gradient are zero at these boundaries. Two possibilities are used for the concentration: concentrations are set equal or large reservoirs with intense mixing are assumed. The no-slip boundary condition and the absence of deposition is assumed near the bodies. Experiments, however, show that deposition is possible near the bodies and it should be discussed in future studies.

The Soret coefficient is defined as being proportional to the ferroparticle flux through a certain vertical cross-section of the system [2]. The Soret coefficient is constant in a stationary case at any vertical cross-section. The calculations allow to obtain also a resulting pressure difference between the both sides of the grid and the difference of concentrations. Because the flow is present accross the whole area with the varying permeability, the size of the system along the x-axis is chosen larger than the size of the area with the varying temperature. This enables to obtain precise values of the Soret coefficient and osmotic pressure difference.

4. Examples. As a particular example, the relaxation dynamics of the inhomogeneous concentration without bodies is considered, where the interplay between the magnetic, flow and concentration fields is very strong. As can be seen in Fig. 1*b*, the region with a higher concentration stretches along the magnetic field lines, whereas in the perpendicular direction the region shrinks. More inter-





a *b*

Fig. 3. (*a*) Development of the concentration and pressure difference at the both sides of the grid with elliptic elements at $B = 0.0375$ T. (*b*) Stationary difference of concentrations at the both sides of the grid versus the magnetic field.

esting effects are possible at stronger magnetic fields and in an axially symmetric coordinate syste, where the area with a higher concentration can split in several filaments.

The interesting dynamic phenomena of fluid in the presence of bodies are even enhanced especially at high intensity of the magnetic field due to a considerable convection. Here we will analyze only the flow behavior at moderate field values. Fig. 2, 3 shows the case when the grid elements are ellipses and the magnetic field is oriented along the temperature gradient. As can be seen from Fig. 2*a*, the flow of ferrofluid decays very slowly in the horizontal direction. Fig. 2*b* shows that area with a higher concentration is located in the grid plane. If the magnetic field is oriented along the y-axis, the areas with higher and lower concentrations would be opposite. The temperature difference between the both sides is 10 K and the temperature is varying within the 1 mm long spatial distance. In case of magnetic field orientation along the x-axis, the ferroparticles are transferred toward higher temperatures, as can be seen from Fig. 3*a*, i.e., the resulting Soret coefficient is negative. The stationary state is achieved within some days, as the directed ferroparticle flow is relatively weak. Fig. 3*b* shows that the equilibrium concentration and the pressure difference between the both sides become saturated at $B \approx 0.05$ T. The Soret coefficient can be calculated if the concentrations of ferrofluid at the both sides of the grid are set equal.

5. Conclusions. Interesting coupling phenomena of the magnetic, concentration, and flow fields takes place in case of ferrofluid around a non-magnetic body in the presence of both magnetic field and temperature gradient. Particular attention in considering the Soret effect and the osmotic pressure difference should be paid to strict maintenance of the mass conservation laws, e.g., solving the system by finite volume methods. The problem related to the convergence of flow far from the grid is solved by setting the area with the temperature gradient finite. The work has been made by support of IPUL-MHD project of the 5th framework.

REFERENCES

1. T. Volker, S. Odenbach. Thermodiffusion in magnetic fluids. *J. Magn. Magn. Mat.*, vol. 289 (2005), pp. 289–291.
2. E. Blums. New problems of particle transfer in ferrocolloids: Magnetic Soret effect and thermoosmosis. *Eur. Phys. J. E*, vol. 15 (2004), pp. 271–276.
3. A. Lange Kelvin force in a layer of magnetic fluid. *J. Magn. Magn. Mat.*, vol. 241 (2002), pp. 327–329.