CONDUCTING WALLS MHD ALTERNATE GENERATOR AT A MODERATE MAGNETIC REYNOLDS NUMBER

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Introduction The work is focused on an oscillating flow in a very long MHD channel of rectangular cross-section and thin conductive walls (Fig. 1). The length L_B of the channel is much higher than the other dimensions, width $2l$ and thickness $2a$, so it can be considered as infinitely long. The wall's thickness $e_{\rm w}$ of this channel and its electrical conductivity $\sigma_{\rm w}$ are such than the thin walls hypothesis can be assumed [1]. An incompressible liquid metal of density ρ, dynamic viscosity μ, magnetic permeability μ_0 and electrical conductivity σ flows in the channel along the x-direction with a velocity amplitude u_0 and a pulsation ω imposed along the channel by an oscillatory pressure gradient with an amplitude $\Delta p/L_B$. The oscillating pressure gradient is obtained by a thermo-acoustic effect that has the potential of producing mechanical power from a heat source with no moving part in a confined container. The principle is based on the use of a temperature gradient imposed at both extremities of a stack of plates properly disposed in a closed tube to create spontaneously a standing wave in the tube ([2] to [9]). The oscillating pressure at both extremities of the MHD channel subjected to a constant external magnetic field $\mathbf{B}_0 = B_0 \cdot \mathbf{e}_y$, imposed by an inductor, generates an AC electric current resulting from the interaction of the imposed magnetic field with the velocity field and collected by two electrodes placed in $z = \pm l$ (Fig. 1); the electric current supplies a load assimilated here to a resistance R.

1. Formulation of the problem. The imposed pressure distribution all along the channel, which is the motive term, is assumed to be of the form

$$p = p_0 + \mathrm{Re}\left(\frac{\Delta p}{L_B} x \cdot \mathrm{e}^{i\omega t}\right) = p_0 + \mathrm{Re}\big(p_1 \cdot \mathrm{e}^{i\omega t}\big)$$

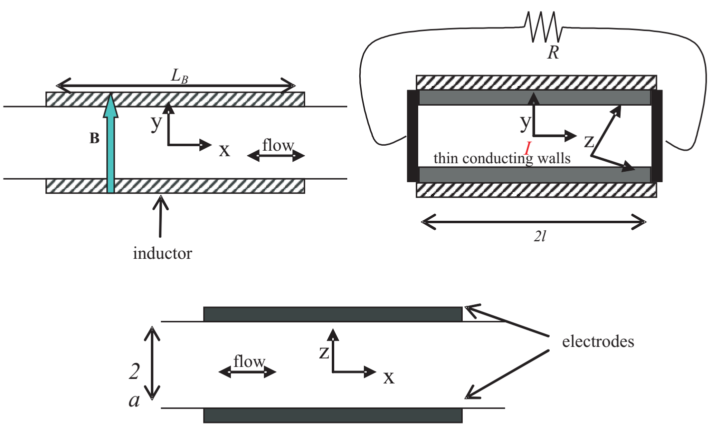

Fig. 1. Views of the MHD channel, the electromagnetic inductor creates a vertical magnetic field, the current lines resulting from the interaction between the magnetic field and the flow are collected by the electrodes.

Complex numbers are only used by commodity, only the real part will be conserved as physical meaning. The other variables are supposed to have the same pulsating form

$$u = R(u_0 \mathrm{e}^{i\omega t})$$
$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b} = \mathbf{B}_0 + \mathrm{Re}(b\mathrm{e}^{i\omega t})$$

With subscript o for the mean value. The mean value of the velocity is zero. The induced magnetic field b is the perturbation of the imposed magnetic field. By choosing the following characteristic: scales: a for the lengths, B_0 for the magnetic field, $1/\omega$ for the time, ωa for the velocity, the scale for the electric current density becomes $\dfrac{B_0}{\mu_0 a}$ and the typical electric field $\omega a B_0$, consequently the typical scale of the current is $\dfrac{B_0 a}{\mu_0}$ and the electric resistance of the unit length of the channel $\dfrac{1}{\sigma a}$ allows to define the typical scale of electrical power $\dfrac{B_0^2 a}{\mu_0^2 \sigma}$. Taking into account the main hypothesis and introducing the non dimensional variable in the Navier–Stokes and linerised induction equations gives the dimensionless governing equations

$$iu'^* = -K_p + \frac{1}{\mathrm{Re}_\omega}\frac{\partial^2 u'^*}{\partial y^{*2}} + \frac{N}{\mathrm{Rm}}\frac{\partial b'^*_x}{\partial y^*} \qquad ib'^*_x = \frac{\partial u^*}{\partial y^*} + \frac{1}{\mathrm{Rm}}\frac{\partial^2 b'^*_x}{\partial y^{*2}}$$

with $K_p = \dfrac{\Delta p}{L_B \omega^2 \rho a}$ representing the dimensionless imposed pressure, u^* and b^* respectively the dimensionless axial component of the velocity and induced magnetic field. The main parameters, which control the phenomena, are: the Reynolds number, $\mathrm{Re}_\omega = \dfrac{\rho\omega a^2}{\mu}$, the magnetic Reynolds number $\mathrm{Rm} = \mu_0 \sigma \omega a^2$, the Hartmann number $\mathrm{Ha} = B_0 a \sqrt{\dfrac{\sigma}{\mu}}$, the interaction parameter $N = \dfrac{\mathrm{Ha}^2}{\mathrm{Re}_\omega} = \dfrac{B_0^2 \sigma}{\omega \rho}$.

2. Boundary conditions. The non slip conditions and symmetry of velocity and induced magnetic field gives:

$$u^*(y^* = 1) = 0, \qquad \frac{\partial u^*}{\partial y}(y = 0) = 0, \qquad b^*_x(y = 0) = 0$$

A fourth condition is needed to specify the global solution. This last condition results in closing of current density lines partially in the wall, partially in the fluid flow and partially in the load resistance. Considering that the electric current circulating channel, and taking into account the continuity of the electric current which closes partially in the resistance, partially in the fluid and in the resistance is imposed by the voltage drop at the both sides of the rectangular partially in the walls, $I^* + I^{*'} + i^* = 0$, provides the fourth boundary condition.

$$b^*_x(y^* = 1) = \frac{K-1}{K}\frac{\partial b^*_x}{\partial y^*}\big(y^* = 1\big), \quad K = \frac{1}{1 + \frac{r}{R} + \frac{r}{r_\mathrm{w}}} \quad \text{is the load factor.}$$

In this expression the ratio $C = r/r_\mathrm{w}$ represents the conductance ratio of the fluid and walls.

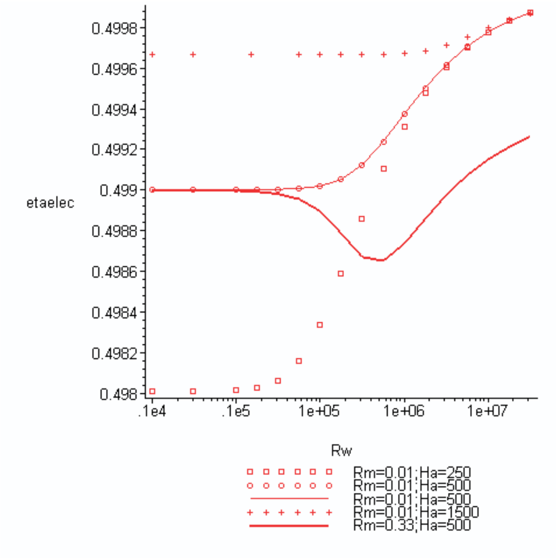

Fig. 2. Electric efficiency versus the Reynolds number for Rm = 0.01 and Rm = 0.33, insulating walls, $K = 0.5$ and three values for Ha. The curve form small Rm and Ha = 500 (circles) fit well the Ibanez work] (thick line). The curve with Rm = 0.33 (bold line) presents a decreasing of efficiency around $N = 1$ at the transition between the Hartmann regime and the inertial regime.

3. Results and discussion. The problem admits an analytical solution which is not given here due to the size and complexity of the two expressions of the current density and velocity distribution. In the asymptotic case of high interaction parameter and sufficiently far from the wall the two fields can be described by a simplified expression:

$$u'^* = \frac{K_p}{i + N(1-K)}, \qquad b'^*_x = \frac{K_p \mathrm{Rm}(1-K)}{i + N(1-K)}$$

The efficiency corresponds to the electric power extracted from the load reported to the mechanical energy introduced in the system by the work of the pressure forces. Since the electric circuit is assimilated to a load resistance R, the electrical power P_e created to supply this circuit is simply the power dissipated by the Joule effect in the resistance. The total efficiency of the system is then defined by:

$$\eta_e = \frac{P_e}{P_e + P_1 + \cdots + P_n},$$

where $P_1 \ldots P_n$ represents the sum of the losses due to the Joule effect and viscous dissipation. The analytic solution allows to calculate all the properties of the flow including energetical aspects. The comparison between the results of the present work with the Ibanez's work [11] reveals a very good agreement. It can be observed, for example, in Fig. 2 that the evolution of the efficiency versus the Reynolds number Re_ω for small Rm, $K = 0.5$, Ha = 500 and insulating walls is very close to the Ibanez results. It can be seen that the efficiency is strongly influenced by the magnetic Reynolds number and Fig. 3 shows that the efficiency decreases when the conductance ratio increases.

4. Conclusion. This study has been focused on the characterisation of an alternate MHD generator resulting from the use of the thermo-acoustic effect to impose an oscillating motive pressure gradient. The effect of the wall conductivity has been taken into account and the solution searched for small or moderate values of the magnetic Reynolds number. The role of the main parameters that control the efficiency, i.e., the interaction parameter, the Reynolds number, the magnetic Reynolds number, the load factor and the conductance ratio have been analysed. The optimisation of the system is very hard and requires a special study.

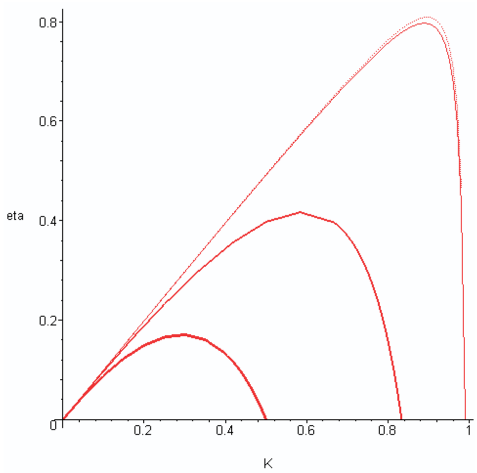

Fig. 3. Efficiency of the system as a function of load factor K for four conductance ratios (increasing thickness), $C_w = 0$, $C_w = 0.01$, $C_w = 0.2$ and $C_w = 1$. Dash line represents the electric efficiency for $C_w = 0$.

Nevertheless, some indications can be given; the best efficiency is obtained for a relatively high value of the load factor ($K \approx 0.8$), moderate value of the interaction parameter in the inertia regime $N > 1.5$, and for the value of the magnetic Reynolds number close to 1. It is important to notice here the influence of this last parameter generally neglected in most of the studies.

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