JOULE HEATING EFFECT IN MHD DUCT FLOWS

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Introduction. Heat transfer in MagnetoHydroDynamic (MHD) flows is important for many applications, e.g. for liquid-metal blankets and divertors for fusion reactors [1, 2, 3, 4, 5]. Because of the limitations of the actual material properties and the efficiency of the electricity production, most Li and PbLi blankets must operate within the relatively narrow range of temperature variation of between 100°C and 200°C [6].

Heat and mass transfer in MHD flows has been analysed in [1, 2, 3, 4]. In all of these studies the associated Joule heating owing to currents induced by the fluid flow has been neglected. This is adequate for electrically insulating ducts or for relatively weak magnetic field characteristic for many heat-transfer laboratory experiments. It has been suggested in [7], however, that the Joule heating in blankets employing ducts with thin conducting walls may be significant. The estimate of the effect in [7] has been made for the Hartmann flow. It has been shown that Joule heating in the thin conducting walls results in significant increase in temperature at the fluid-wall interface and at the outer wall surface.

Here we evaluate the Joule heating for the flow in a rectangular duct with thin conducting walls and compare the results with those obtained earlier for the Hartmann flow. It is shown that the intensity of the Joule heating depends strongly on the wall conductance ratio, the velocity profile, the magnitude of the magnetic field, etc., and imposes limits on all the above flow parameters.

1. Formulation. Consider the steady, fully developed flow of a viscous, electrically conducting, incompressible fluid along a straight, rectangular duct in the presence of an applied magnetic field $\mathbf{B} = B_0^*\hat{\mathbf{y}}$. The flow is driven by the pressure gradient imposed in the x-direction. The distance between the Hartmann walls is $2a^*$, while the thickness of the wall is h_w^*.

The dimensionless equations governing the flow are [5]:

$$\mathrm{Ha}^{-2}\nabla^2\mathbf{v} + \mathbf{j}\times\mathbf{B} = -\nabla p, \qquad \mathbf{j} = -\nabla\phi + \mathbf{v}\times\mathbf{B}, \qquad \nabla\cdot\mathbf{v} = 0, \qquad \nabla\cdot\mathbf{j} = 0,$$
$$\mathrm{Pe}(\mathbf{v}\cdot\nabla)T = \nabla^2 T + \mathbf{j}^2, \qquad \lambda_\mathrm{w}\nabla^2 T_\mathrm{w} + \sigma_\mathrm{w}^{-1}\mathbf{j}_\mathrm{w}^2 = 0, \qquad \mathbf{j}_\mathrm{w} = -\sigma_\mathrm{w}\nabla\phi_\mathrm{w}\,.$$

Here the length, the fluid velocity $\mathbf{v} = u\hat{\mathbf{x}}$, the pressure p, the electric current density $\mathbf{j}$, the electric potential ϕ, and the temperature T are normalized by a^*, ν_0^* (average flow velocity), $a^*\sigma^*\nu_0^*B_0^{*2}$, $\sigma^*\nu_0^*B_0^*$, $a^*\nu_0^*B_0^*$ and $(T^* - T_0^*)/\Delta T^*$, respectively. The characteristic temperature difference $\Delta T^* = a^{*2}\sigma^*\nu^{*2}B^{*2}/\lambda^*$ is based on the Joule dissipation. Subscript w refers to wall quantities. The square of the Hartmann number, $\mathrm{Ha}^2 = a^{*2}B_0^{*2}\sigma^*/\rho^*\nu^*$, characterizes the ratio of the electromagnetic to the viscous forces. The Peclet number, $\mathrm{Pe} = \rho^*C_p^*\nu_0^*a^*/\lambda^*$, expresses the ratio of the convective and conductive heat fluxes. In the above, ρ, ν^*, λ^*, σ^* and C_p^* are the fluid density, kinematic viscosity, thermal conductivity, electrical conductivity and specific heat of the fluid, respectively; $\lambda_\mathrm{w} = \lambda_\mathrm{w}^*/\lambda^*$ and $\sigma_\mathrm{w} = \sigma_\mathrm{w}^*/\sigma^*$ are dimensionless thermal and electrical conductivities of the walls.

The boundary conditions at the fluid-wall interface are =0, $\mathbf{j}\cdot\hat{\mathbf{n}} = c\nabla_\mathrm{w}\phi_\mathrm{w}$ and continuity of temperature and heat fluxes. Here $\hat{\mathbf{n}}$ is a unit vector normal to

Table 1. Material properties of Li, PbLi^{17} and stainless steel.

	σ^* $(10^{-6}\,\mathrm{Ohm}\cdot\mathrm{m})^{-1}$	λ^* (W/mK)	ρ^* (kg/m^3)	ν^* $(10^{-6}\,m^2/s)$
Li	3.3434	40.64	504.9	0.8911
PbLi^{17}	0.7917	13.184	9491.7	0.2209
Stainless steel	1.25	22.2		

the wall into the fluid, $c = \sigma_\mathrm{w} h_\mathrm{w}$ is the wall conductance ratio, $h_\mathrm{w} = h^*_\mathrm{w}/a^*$ is the dimensionless thickness of the walls. The outside surface of the walls is both electrically and thermally insulating. The temperature at the entrance to the duct is set to zero.

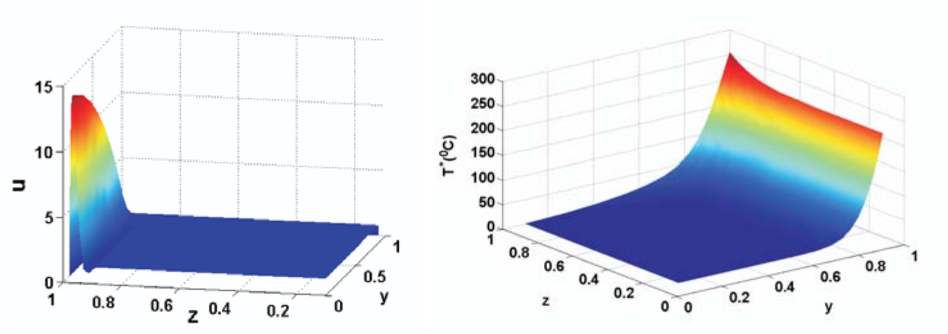
2. Results and discussion. Consider flow of Li or PbLi in a square duct with stainless steel walls. Their properties are listed in Table 1 [5], [9]. Other dimensional quantities are $B^*_0 = 5$ T, $\nu^*_0 = 2$ m/s, $a^* = 0.06$ m, $h^*_\mathrm{w} = 0.003$ m, $l^* = 5.4$ m. Here l^* is the length of the duct, and roughly corresponds to a half-length of the poloidal ducts of a self-cooled blanket. This gives $c = 0.019$, Pe $= 6380$, Ha $= 25859$ for Li and $c = 0.079$, Pe $= 16396$, Ha $= 5820$ for PbLi, respectively.

The velocity and the electric potential are independent of temperature and are obtained from [8]. The energy equation is solved using CFX4.4. Taking advantage of the symmetry, the temperature is calculated in a quarter of the duct cross-section.

Owing to low values of c, most of the Joule dissipation occurs in the conducting walls, which carry all the current induced in the core. The heat diffuses into the duct. As the Peclet number is high, the whole blanket operates in the thermal boundary layer regime; cf. [4]. Heat generated by currents in the fluid is negligibly small.

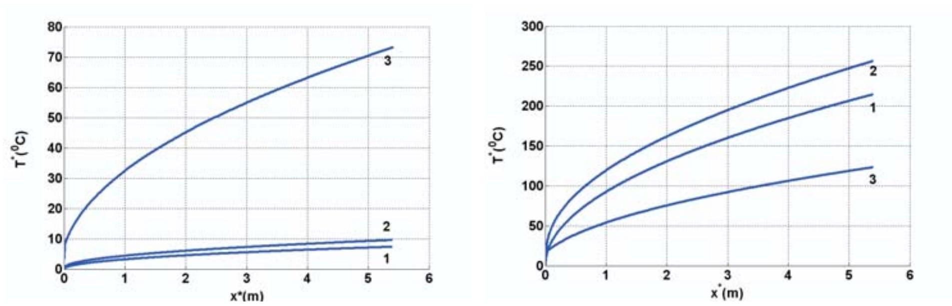
The fluid velocity and temperature for PbLi at $x = l^*$ are shown in Fig. 1. The flow is characterised by high velocity jets near the wall $z = 1$, which enhance the convective heat transfer at the sidewall (Fig. 1a). At the Hartmann wall it is weaker, which leads to high temperature at $y = 1$ (Fig. 1b).

The development of the dimensional temperature with the axial length at the centre ($y = 1$, $z = 0$) and at the corner of the duct ($y = 1$, $z = 1$) is shown in Fig. 2. The dimensional temperature versus the axial length in the Hartmann flow [7] with the same parameters is also shown in the Fig. 2.



(a) (b)

Fig. 1. Dimensionless velocity and temperature at $l^* = 5.4$ m for $c = 0.079$, Pe $= 16396$, Ha $= 5820$.



(a) (b)

Fig. 2. Temperature vs axial length for (a) Li; (b) PbLi. Curves (1) and (2) correspond to temperature for the square duct at $y = 1$, $z = 0$ and at $y = 1$, $z = 1$, resp. Curve (3) corresponds to temperature at $y = 1$ in the Hartmann flow.

From Figs. 1b and 2 follows that the peak in temperature appears at the corner of the duct. The temperature at $y = 1$, $z = 1$ is higher than that at $y = 1$, $z = 0$. The first reason for this is that at the corner the current-generated heat diffuses into the duct from both Hartmann- and side- wall simultaneously. Moreover, high velocity jets near the sidewalls convect heat away from the sidewalls very quickly near the central part of the sidewalls. At the corner of the duct the velocity reduces to values even lower than those in the core region, so that convective heat transfer is weaker.

The rate of increase in temperature along the duct is greatly affected by the wall conductance ratio. For Li with $c = 0.019$, the temperature rise at the corner is only 10°C over the half-length of the blanket. For PbLi[17] with $c = 0.079$, the corresponding temperature rise is 256°C.

The temperature in the square duct has been compared with that in the Hartmann flow. It follows that for Li the temperature rise in the Hartmann flow is much higher than that in the square duct. However, the tendency for PbLi is reversed. This could be explained by two factors, which affect the Joule heating. The first one is the magnitude of the electric current in the thin conducting walls. When the wall conductance ratio is sufficiently low, the electric current in the square duct is lower than that in the Hartmann flow. As the wall conductance ratio increases, the electric current in the square duct becomes almost equal to that in the Hartmann flow. The second factor is the velocity profile. In the Hartmann flow the velocity is almost uniform, $u = 1$. In the square duct part of the volumetric flux is carried by the jets, which leads to a reduced velocity in the core. This implies weaker convective heat transfer away from the jets and higher temperature as a result.

Fig. 3 shows the temperature at the wall for $a^* = 0.06$ m, $\lambda = 1.69$, $h_{\mathrm{w}} = 0.05$, $l^* = 5.4$ m, Pe $= 16396$, Ha $= 5820$; σ_{w} varies between 0.6 and 1.58. The temperature at the fluid-wall interface increases as the wall conductance ratio increases in both square duct and the Hartmann flow. The reason is that the higher wall conductance ratio results in higher electric currents in the thin conducting walls. For $c < 0.055$, the temperature is higher in the Hartmann flow than in the square duct. When $c > 0.055$ the temperature at the corner ($y = 1$, $z = 1$) of the square duct is higher than in the Hartmann flow. When $c > 0.059$, the temperature rise at the centre ($y = 1$, $z = 0$) of the square duct flow also exceeds that in the Hartmann flow. Thus, for the dimensionless parameters used in present calculations, the maximum temperature owing to Joule heating in the thin conducting walls can be estimated by that in the Hartmann flow for $c < 0.055$. For

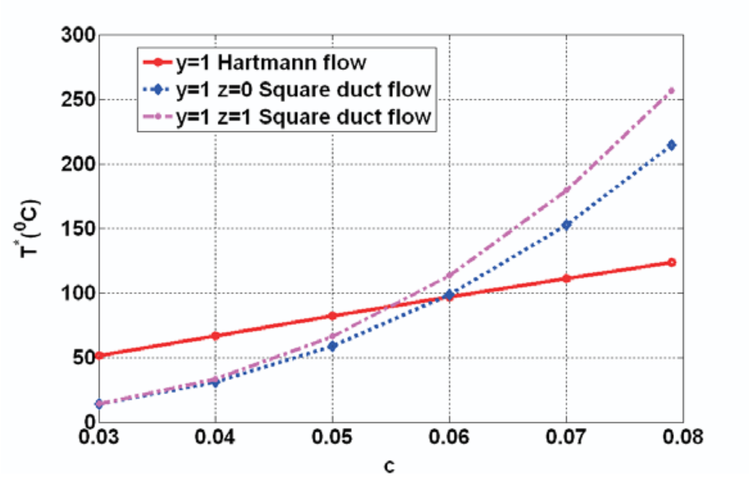


Fig. 3. Dimensional temperature variation with the electric conductivity of the wall in the square duct flow and the Hartmann flow at $l^* = 5.4$ m.

$c > 0.055$, the Hartmann flow approximation is inadequate and the Joule heating effect can only be estimated using the rectangular duct model.

3. Conclusions. Joule heating effect in fully developed liquid metal magnetohydrodynamic flows with square cross section has been investigated numerically. It has been shown that the wall conductance ratio and the velocity profile affect the effect significantly. The Joule heating may well be estimated using the Hartmann flow for relatively low values of the wall conductance ratio. For higher values of c, the effect of the sidewalls on the flow cannot be neglected. Overall, the Joule heating effect may be a significant factor for the development of blankets not employing insulating coatings.

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