THREE-DIMENSIONAL MAGNETOHYDRODYNAMIC FLOWS IN SUDDEN EXPANSIONS

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Introduction. The three-dimensional, inductionless flow of electrically conducting incompressible fluids in symmetric sudden expansions with rectangular cross-section and walls of finite conductivity, exposed to a uniform magnetic field, is investigated numerically using the fluid dynamic code CFX.

The geometry considered is closely related to applications in fusion reactor blankets in which a liquid metal is used as breeder material. The fluid is circulated for removing tritium flowing through manifolds, expansions and contractions.

The present study deals with rectangular channels which expand along the magnetic field lines since this condition creates the strongest MHD interaction (see [1] for flows expanding in the direction perpendicular to the magnetic field). The problem of MHD flows in sudden expansions has been studied experimentally by [2] for small interaction parameters ($N < 2$) and large Reynolds numbers considering ducts with insulating walls. A numerical solution for three-dimensional inertial MHD flow in sudden non conducting expansions under a magnetic field of arbitrary orientation has been described by [3]. The three-dimensional inertial MHD flow in ducts with walls of finite conductivity is still an open problem.

The aim of the present work is to investigate the magnetic influence on the size and the occurrence of the separation regions which form behind sharp edges. Results for a fixed Reynolds number, Re = 100, are shown as a reference case. Starting from the flow pattern for hydrodynamic flow, the Hartmann number, Ha, is gradually increased.

1. Formulation of the problem. Under the assumption of a small magnetic Reynold number, $\mathrm{Rm} = \mu\sigma L v_0 \ll 1$, which implies that the induced magnetic field can be neglected, the dimensionless, inductionless equations governing the steady flow of incompressible electrically conducting fluids under the action of a uniform magnetic field can be written as follows.

$$\frac{1}{N}\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \frac{1}{\mathrm{Ha}^2}\Delta\mathbf{v} + \mathbf{j}\times\mathbf{B}, \quad \nabla\cdot\mathbf{v} = 0, \tag{1}$$

Momentum equation and conservation of mass,

$$\mathbf{j} = -\nabla\phi + \mathbf{v}\times\mathbf{B}, \quad \nabla\cdot\mathbf{j} = 0. \tag{2}$$

In these equations $\mathbf{v}$, $\mathbf{B}$, $\mathbf{j}$, p and ϕ stand for velocity, magnetic field, current density, pressure and electric potential, scaled by the reference quantities v_0, B_0, $j_0 = \sigma v_0 B_0$, $\sigma v_0 B_0^2 L$ and $v_0 B_0 L$, respectively, where B_0 is the magnitude of the externally applied magnetic field and L is the length scale chosen as the half width of the channel in the z direction. The dimensionless groups characterizing the problem are the interaction parameter N, which measures the ratio of the electromagnetic to inertial forces and the Hartmann number Ha, whose square gives the strength of the electromagnetic to viscous forces. In the following discussion the Reynolds number Re is used and it represents the ratio of inertial to viscous forces:

$$N = \frac{\sigma L B_0^2}{\rho v_0}, \quad \mathrm{Ha} = B_0 L \sqrt{\frac{\sigma}{\rho\nu}}, \quad \mathrm{Re} = \frac{\mathrm{Ha}^2}{N}.$$

The fluid properties like the density ρ, the kinematic viscosity ν and the electric conductivity σ are assumed to be constant.

The duct walls have a finite thickness t_{w} and conductivity $\sigma_{\mathrm{w}} \neq 0$. As a consequence part of the current flowing in the fluid may close its path in the walls. The dimensionless equations governing the electric variables in the wall are:

$$\mathbf{j}_{\mathrm{w}} = -\frac{\sigma_{\mathrm{w}}}{\sigma} \nabla \phi_{\mathrm{w}}, \quad \nabla \cdot \mathbf{j}_{\mathrm{w}} = 0. \tag{3}$$

As kinematic boundary conditions at the fluid-wall interface the no-slip condition is applied, $\mathbf{v} = 0$. The electric boundary conditions at the interface state the continuity of electric potential and wall normal component of current density, $\phi = \phi_{\mathrm{w}}$, $j_{\mathrm{n}} = j_{\mathrm{nw}}$.

2. Results and discussion. Let us consider the steady, 3D flow of an electrically conducting fluid flowing in the x direction in a symmetric sudden expansion of aspect ratio 4, exposed to an applied magnetic field, $\mathbf{B} = B\mathbf{y}$ (Fig. 1). The geometry is chosen according to the features of the experimental test section used in the MEKKA laboratory in the Forschungszentrum Karlsruhe. The walls are assumed to have a finite conductivity σ_{w} and thickness t_{w} and the wall conductance parameter $c = \sigma_{\mathrm{w}} t_{\mathrm{w}} / \sigma L$, which describes the ratio of the electrical conductance of the wall and the fluid material is equal to 0.1.

The process of flow separation is analyzed considering a constant Reynolds number, $\mathrm{Re} = 100$, and increasing gradually the Hartmann number.

Fig. 2 shows the profiles of the transverse velocity w plotted along the duct axis in the symmetry plane $y = 0$ for $z = 0.9$ for $\mathrm{Ha} = 10, 50, 100, 350$. The idea is to find a way to describe and visualize the thickness δ of the internal layer that spreads into the fluid along magnetic field lines [4]. Let us consider as an indicator of this thickness the distance from the expansion ($x = 0$) to the point in which the transverse velocity component changes the sign. This assumption is supported by the observation that also the z component of the current and the x component of the pressure gradient present the largest variations within this length. Considering these velocity profiles at different z, it is found that the position of the point of interest varies in a very small range at least for su.ciently high interaction parameter. From Fig. 2 it can be seen that the thickness of the internal layer decreases by increasing the Hartmann number. When the inertial

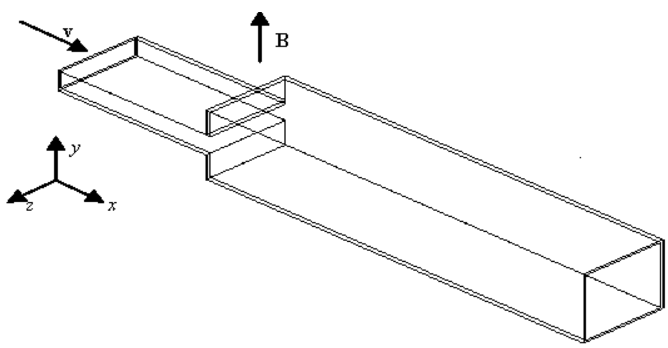

Fig. 1. Sketch of the geometry.

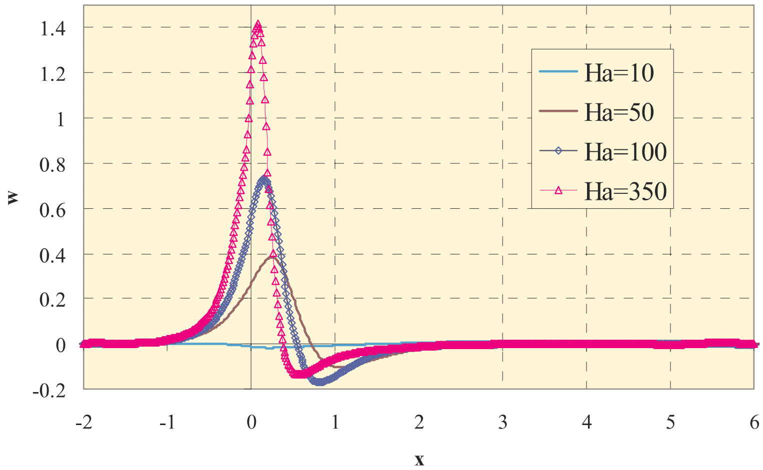

Fig. 2. Velocity component w along x on the symmetry plane $y = 0$ at $z = 0.9$.

e.ects can be neglected, the thickness of the internal layer depends only on the Hartmann number and varies as $\mathrm{Ha}^{1/2}$.

In Fig. 3*a* the extension of the recirculation areas near the Hartmann wall at $y = 0.998$ is shown for Ha = 10, 50, 100 and 350. Fig. 3*b* shows the location of the separation regions close to the side wall at $z = 0.994$ for Ha = 50, 100 and 350. In the hydrodynamic flow (Ha = 0) for the same Reynolds number, two asymmetric separation zones occur and the longest extends up to $x = 14$. For Ha = 10 two symmetric recirculations are present and their length reaches $x = 3.8$. For $Ha \geq 50$ there are four separated recirculations located in the duct corners and the separation length is further reduced. Therefore it can be asserted that the application of a magnetic field results in damping the separation effect.

Results for velocity profiles at different axial positions in the symmetry plane $y = 0$ and at $y = 0.75$ are shown in Fig. 4 for Ha = 100. Downstream and upstream the fully developed flow profile is established. In Fig. 4*a* it can be seen that, behind the expansion, the velocity increases in the side layers. At $y = 0.75$ (Fig. 4*b*) immediately after the expansion near the corners, there is a reversed flow.

The numerical results for high interaction parameters are in agreement with the asymptotic theory [5].

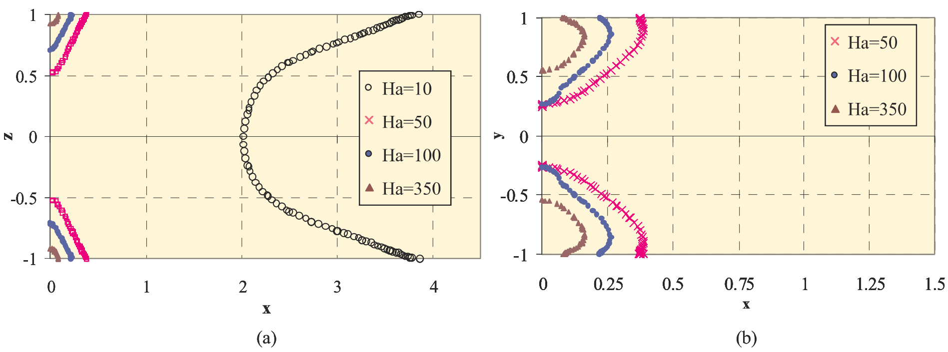

Fig. 3. Extension of recirculation zones close to the upper wall of the duct at $y = 0.998$ (a) and near the lateral wall at $z = 0.994$ (b).

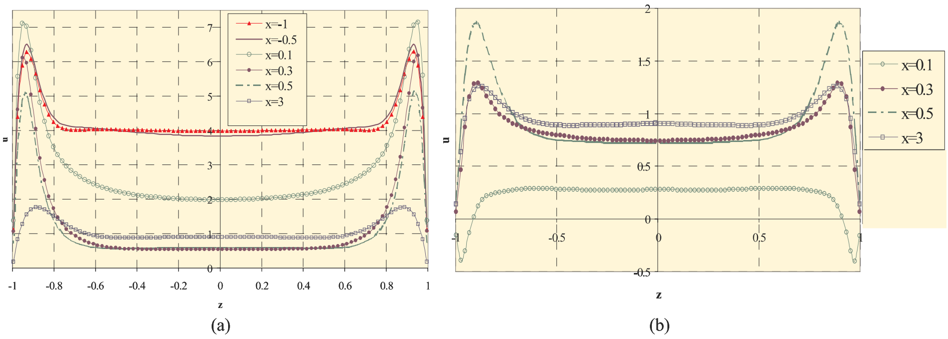

Fig. 4. For Ha = 100 and Re = 100 the u velocity is plotted along z for various axial positions (a) on the symmetry plane at $y = 0$ and (b) at $y = 0.75$.

3. Conclusions. The effect of a uniform transverse magnetic field on liquid metal flow in rectangular ducts with sudden expansions has been investigated numerically using the commercial code CFX. The duct walls are assumed to have finite conductivity and the conductance parameter is $c = 0.1$. Changing the Hartmann number in the range $10 \leq Ha \leq 350$, while keeping the Reynolds number constant at Re = 100, the structure and the size of the separation regions that form behind the expansion have been analyzed. In the hydrodynamic case (Ha = 0) at Re = 100 two asymmetric recirculations occur and the largest one extends up to $x = 14$. By increasing the Hartmann number the separation regions becomes symmetric and smaller. For Ha ¿ 50 there are four separated recirculations in the corner of the duct. For Ha 10 two symmetric separation areas were found. The three dimensional MHD flow for Ha $\geq$ 400 is currently under investigation with the purpose to study the flow in the range of values covered by the experiments conduced at the Forschungszentrum Karlsruhe.

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