NUMERICAL COMPUTATIONS FOR A FREE-SURFACE FLOW OF LIQUID METAL UNDER A UNIFORM MAGNETIC FILED

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Transient process of a liquid metal column break in the presence of a uniform magnetic field has been solved numerically with a new-developed numerical method for two-phase flows. A rectangular liquid metal column, which is located at a corner of a rectangular enclosure, is abruptly released in the gravitational field and it spreads freely in the enclosure. In this paper, the effect of a uniform magnetic field, which is longitudinal, transverse, or vertical, has been studied for the liquid metal column break. Depending on the strength and the direction of an applied magnetic field, the transient flow exhibits significant difference. When it is applied in the longitudinal or transverse direction, the damping effect of magnetic field on the spreading flow is not strong. However, when it is in the vertical direction, the rate of spreading flow is substantially reduced. The new numerical method enables us to make a two-phase flow like this liquid metal break flow quite easily.

Introduction. Two-phase flows have been studied for their importance in various fields of industry and metal processing. However, those flows encountered in such a metal processing are difficult to visualize and detect the flow motion inside the molten metal. Therefore, numerical models to simulate such two-phase flows of liquid metal are required. Molokov [1] studied a dynamics of a free-surface flow in a strong vertical magnetic field with an asymptotic analysis for high Hartmann number. Ueno *et al.* [2] performed a numerical simulation of a deformed bubble rising in a magnetic fluid in the presence of a vertical magnetic field. Tagawa [3] carried out a numerical computation of spin-up from rest in a uniform axial magnetic field with taking deformation of free surface into account. In the present study, as a representative problem of free-surface flows, transient process of a liquid metal column break in the presence of a uniform magnetic field is numerically simulated in order to show the validity of a new modeling of MHD two-phase flow.

1. Model considered. Let us consider a fundamental problem of a free-surface flow of liquid metal. In a rectangular enclosure of an aspect ratio 2:1:1, a liquid metal column is initially located at one of four corners and it is abruptly released in the gravity field. The liquid metal column immediately spreads freely in the rectangular enclosure but its motion is affected by the Lorentz force due to an applied magnetic field as shown in Fig. 1. A uniform magnetic field is imposed to investigate the effect of its direction on fluid motion. They are applied in longitudinal (x-direction), transverse (y-direction) or vertical (z-direction). In

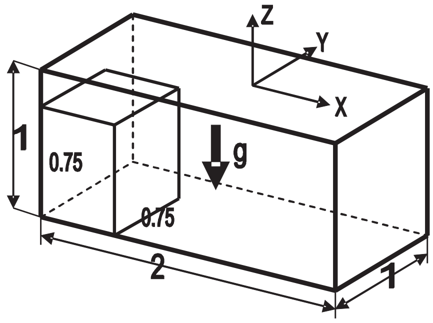


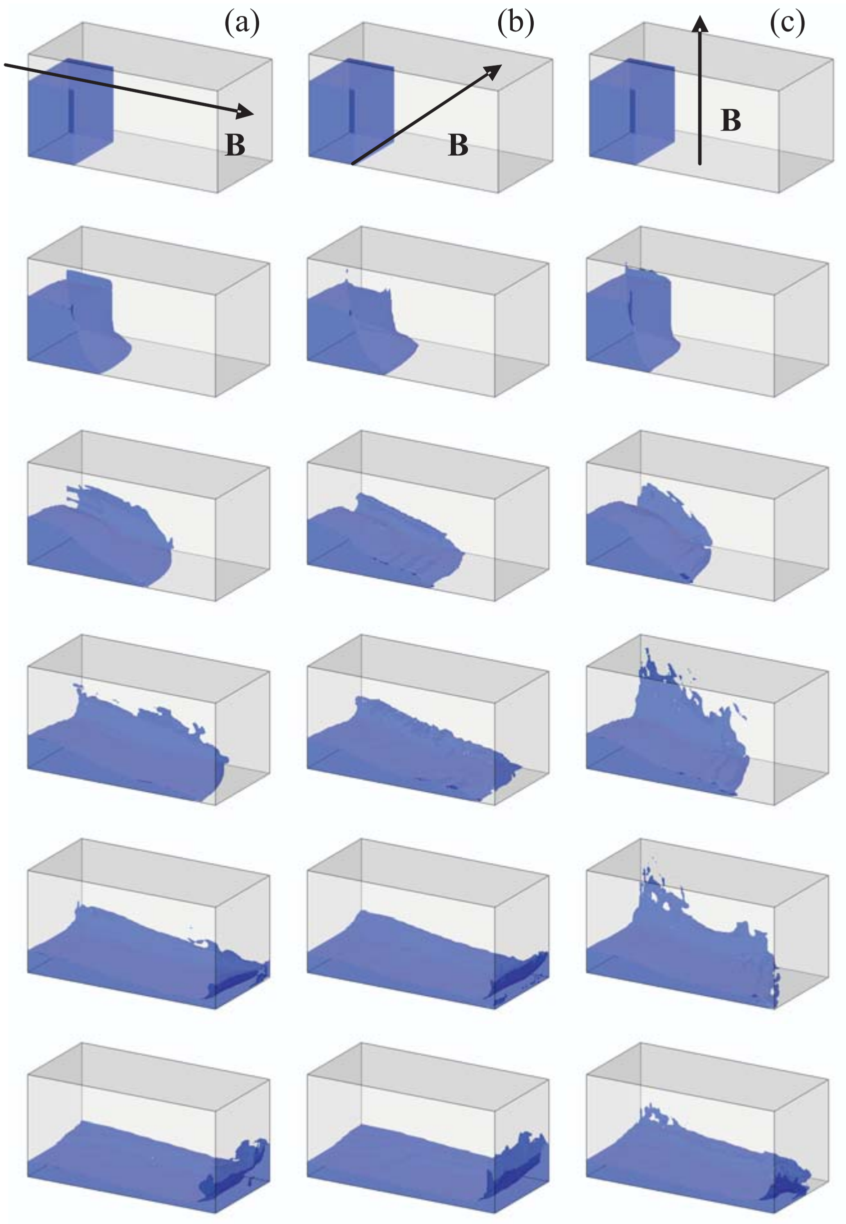
Fig. 1. Schematic model considered.

this paper, the Joule heating and the induced magnetic field are neglected and an isothermal state is considered. The fluid is incompressible Newtonian for both liquid and gas phases. The enclosure walls are electrically insulating and therefore no electric current penetrates into the walls.

2. Equations. The non-dimensional modeled equations are summarized as follows:

$$\frac{\partial \mathbf{U}}{\partial \tau} + (\mathbf{U}\cdot\nabla)\mathbf{U} = -\frac{1}{\rho_\phi}\nabla P + \frac{\mu_\phi}{\rho_\phi}\nabla^2\mathbf{U} + \frac{\widehat{\mu}}{\rho_\phi}\cdot \mathrm{Ha}^2\mathbf{J}\times\mathbf{B} - \mathrm{G}\mathbf{e}_z$$

$$\nabla\cdot\mathbf{U} = 0, \quad \mathbf{J} = \sigma_\phi(-\nabla\Psi + \mathbf{U}\times\mathbf{B}), \quad \nabla\cdot\mathbf{J} = 0, \quad \frac{\partial\phi}{\partial\tau} + (\mathbf{U}\cdot\nabla)\phi = 0$$



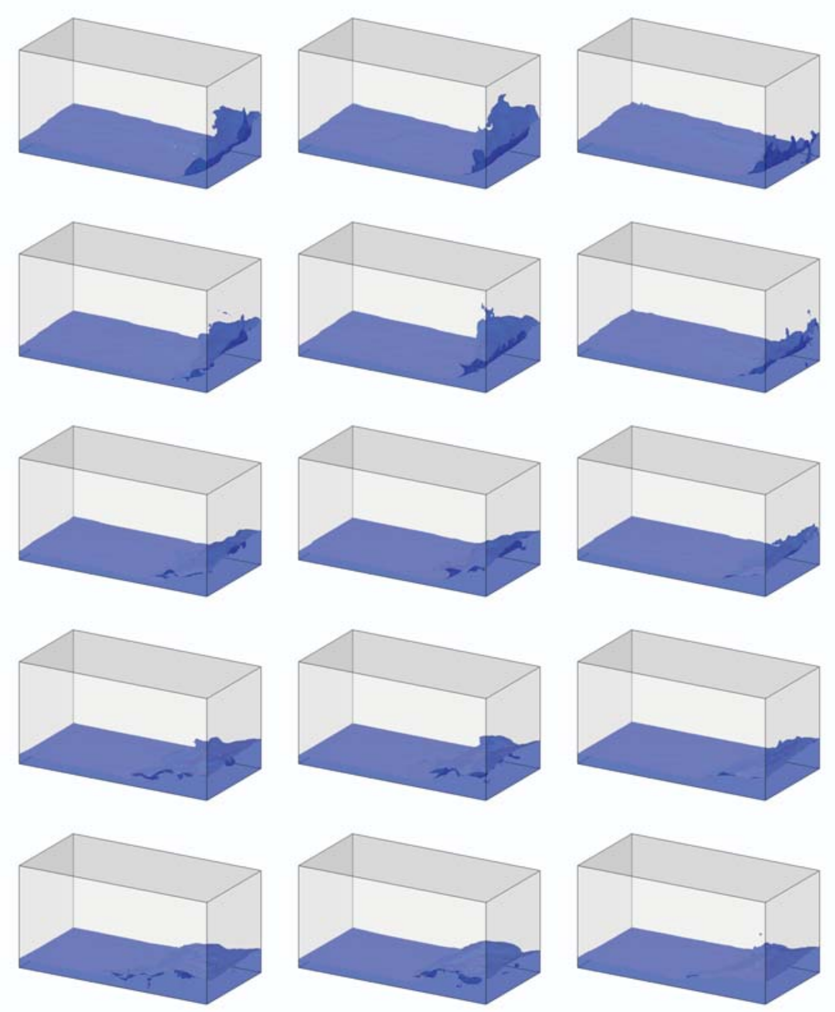


Fig. 2. A sequence of flow for (*a*) x-, (*b*) y-, and (*c*) z-directional magnetic fields. G = 106, Ha = 100, the density ratio $\widehat{\rho}$, the viscosity ratio $\widehat{\mu}$ and the conductivity ratio $\widehat{\sigma}$. Dimensionless time interval is 0.0005.

The interfacial properties such as density, viscosity and electric conductivity are defined as follows:

$$\rho_\phi = \frac{\rho}{\rho_\mathrm{G}} = \frac{1}{2}\left(\frac{\rho_\mathrm{L}}{\rho_\mathrm{G}} + 1\right) - \left(\frac{\rho_\mathrm{L}}{\rho_\mathrm{G}} - 1\right)\frac{1}{2}\sin\left(\pi\,\phi\right)$$

$$\mu_\phi = \frac{\mu}{\mu_\mathrm{G}} = \frac{1}{2}\left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{G}} + 1\right) - \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{G}} - 1\right)\frac{1}{2}\sin\left(\pi\,\phi\right)$$

$$\sigma_\phi = \frac{\sigma}{\sigma_\mathrm{L}} = \frac{1}{2}\left(1 + \frac{\sigma_\mathrm{G}}{\sigma_\mathrm{L}}\right) - \left(1 - \frac{\sigma_\mathrm{G}}{\sigma_\mathrm{L}}\right)\frac{1}{2}\sin\left(\pi\,\phi\right)$$

Where, Ha represents the Hartmann number, and G represents the Galilei number. In the present study, surface tension is neglected. In present numerical model, three physical properties such as density ratio $\widehat{\rho}$, viscosity ratio $\widehat{\mu}$ and electric conductivity ratio $\widehat{\sigma}$ are relevant parameters.

3. Numerical strategy. The above set of equations are transformed to finite difference equations on a staggered-mesh system and solved numerically with a finite difference method using the HSMAC algorithm [4]. In order to reduce the numerical diffusion, a third-order upwind scheme is utilized for the advective terms of equations. Besides, the method of the transformation of tangent function [5] is introduced. This method is

$$H = \tan\left(c\pi\phi\right), \quad c = 0.9.$$

The value of c can be arbitral but it should be nearly equal to unity. The advective equation for ϕ becomes

$$\frac{\partial H}{\partial \tau} + \left(\mathbf{U}\cdot\nabla\right)H = 0.$$

After having got a distribution of H within the computational domain, the inverse transformation of tangent function is made to get a distribution of ϕ. It is given as

$$\phi = \tan^{-1}\left(H/c\pi\right), \quad c = 0.9.$$

By this simple technique, the thickness of the interface is kept thin quite easily. The number of meshes is $120\times60\times60$ in the x-, y-, and z-directions respectively.

4. Computational results. Figure 2 shows a sequence of results for the three cases. In (a), the uniform magnetic field is in the x-direction while in (b) and (c), the uniform magnetic field is applied in the y-direction or z-direction respectively. The liquid metal column breaks due to its weight just after sudden imposition of the gravity filed but its movement is simultaneously influenced by the Lorentz force when liquid conductor moves in a magnetic field. From Fig. 2, it can be recognized that the magnetic field in the z-direction has a most powerful effect on the liquid metal to reduce the flow intensity.

5. Conclusion. Numerical computations were performed for a liquid metal column break in the presence of a uniform magnetic field with a new-developed numerical model of MHD free-surface flow. The present results exhibit significant difference in the column break depending on the direction of an imposed uniform magnetic field. This numerical model of the two-phase flow can be applied to other problems such as droplet flow, bubble flow and non-Boussinesq natural convection.

REFERENCES

1. S. Molokov. Evolution of a free surface in a strong vertical magnetic field. In *Proceedings of the Fifth International Pamir Conference on Magnetohydrodynamic* (Ramatuelle, France, 2002), vol. 2, pp. 65–70.

2. K. Ueno, T. Nishita, S. Kamiyama. Numerical simulation of deformed single bubbles rising in magnetic fluid. *Journal of Magnetism and Magnetic Materials*, vol. 201, (1999), pp. 281–285.

3. T. Tagawa. Spin-up from rest in a cylinder in a uniform axial magnetic field: deformation of free surface. In *Proceedings of Japan-France Cooperative Science Program Seminar on Evolving New Fields in Electromagnetic Processing of Materials* (Keihanna, Japan, 2004), pp. 101–104.

4. C.W. Hirt, B.D. Nichols, N.C. Romero. A numerical simulation algorithm for transient fluid flow. (Los Alamos Scientific Laboratory Report, LA-5852, 1975).

5. T. Yabe, T. Utsumi, Y. Ogata. CIP method (Morikita shuppan Co., Ltd., 2003). (in Japanese).