ELECTRICALLY DRIVEN VORTICES IN A NON-HOMOGENEOUS MAGNETIC FIELD IN SHALLOW FLUID LAYERS

E. Ramos[1], *F. Demiaux*[2], *S. Cuevas*[1], *H. Salas*[1]

[1] *Center for Energy Research, National University of Mexico, A. P. 34, Temixco, Mor. 62580 Mexico (erm@cie.unam.mx)*

[2] *INSA-Lyon, 20 Av. Albert Einstein, 69621 Villeurbanne Cedex, France*

Introduction. Flows in shallow layers of liquid have been a convenient model to analyze two-dimensional turbulence and vortex flows. In many studies, the motion on the fluid is promoted by the Lorentz force which results from the interaction of an electric current and a magnetic field. In these experiments, the working fluid must be electrically conducting, typically a liquid metal or an electrolyte. Important features of the two-dimensional flows that have been studied in thin layers of liquids include the inverse cascade typical of two-dimensional turbulence [1] and chaotic fluid mixing due to the interaction of vortical structures generated by periodic magnetic forcing [2]. A specific important feature of the flow in shallow layers is to determine the conditions under which this flow is two-dimensional. Detailed studies in the context of turbulent dissipation have appeared in the literature and now it is clear that the flow in a density stratified layer does display the expected properties of two-dimensional turbulence, as described, for instance, in [3]. However, it is not clear how the transversal motion disrupts the two-dimensional behavior of the flow in magnetohydrodynamic forced flows in homogeneous layers. The present investigation is a contribution to clarify this issue. In the present paper we report a theoretical and experimental investigation on the laminar flow generated by nonhomogeneous electromagnetic forces in a thin layer of an electrolyte. Particle Image Velocimetry results were obtained in planes of motion parallel to the bottom wall at different depths as well as in planes transversal to this wall, but only the permanent regime of the forced vortex motion was analyzed. A numerical solution of a two-dimensional magnetohydrodynamic equations was developed and a good agreement with experimental results is obtained.

1. Experimental setup. The experimental setup consists of an open rectangular container made of acrylic and glass with internal dimensions $36 \times 28 \times 1.3$ cm, with its larger wall lying horizontally. A cylindrical Neodynium Iron Boron magnet (1.9 cm in diameter) is fixed underneath the bottom wall of the container at its geometrical center. The magnetic field at the center of the magnet points in the positive vertical direction and is perpendicular to the bottom of the container. The maximum magnetic field measured at the center of the magnet is 3300 Gauss and reduces to less that 5% of its maximum value at a distance of one diameter. One of the large lateral walls is made of glass to allow the laser beam sheet to enter horizontally with minimum attenuation and scatter. The working fluid is a water solution of sodium bicarbonate ($NaCO_3H$) at 8.8% in weight. The solution has a density of 1086 kg/m^3 and a kinematic viscosity of 10^{-6} m^2/s. Two electrodes made from copper bars are placed next to each of the smaller sides of the container and connected to an external power source. The current across the layer of electrolyte was varied in the range 10–100 mA. A Particle Image Velocimeter

Fig. 1. Photograph of the experimental set up.

consisting of a Yag laser, a video camera and electronics to synchronize the laser and the camera and process the information is installed to measure the velocity in horizontal planes at different depths. The camera is mounted at the top of the container and looks downwards as shown in Fig. 1, where a picture of the experimental equipment is presented.

2. Theoretical Model. We consider a thin (depth << length, width) layer of an incompressible, Newtonian electrolyte with density ρ, kinematic viscosity ν and electric conductivity σ. The magnetic field can be written as $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, where $\mathbf{B}_o$ and $\mathbf{b}$ are the applied and induced magnetic fields, respectively. Given the geometry of the fluid layer, the motion is mostly confined to the horizontal plane and we propose a two dimensional model with velocity $\mathbf{u} = (u(x,y,t), v(x,y,t), 0)$, externally imposed magnetic field $\mathbf{B}_0 = B_z^0(x,y)\mathbf{k}$, induced magnetic field $\mathbf{b} = b_z(x,y)\mathbf{k}$ and electric current $\mathbf{j} = (-1 + j_x^i)\mathbf{i} + j_y^i\mathbf{j}$. Note that we have written this last variable as the sum of the imposed elctric current in the direction $-\mathbf{i}$ and the induced current $j_x^i\mathbf{i} + j_y^i\mathbf{j}$.
The nondimensional governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} \tag{3}$$

$$0 = \nabla_\perp^2 b_z - u\frac{\partial B_z^0}{\partial x} - v\frac{\partial B_z^0}{\partial y}, \tag{4}$$

where

$$\mathrm{Re} = \frac{j_0 B_m D^2/\rho\nu}{\nu/D} = \frac{U_0}{u_0} = \frac{U_0 D}{\nu}, \qquad \text{and} \qquad U_0 = \frac{j_0 B_m D^2}{\rho\nu}. \tag{5}$$

The characteristic velocity U_0 is established from the balance of viscous effects and Lorentz force $|\nu\nabla^2\mathbf{u}| \sim |\mathbf{j}\times\mathbf{B}_0/\rho|$. B_m and D are, respectively, the diameter of the magnet and the magnitude of the maximum magnetic field externally imposed.The symbol $\nabla_\perp^2$ stands for $\partial^2/\partial x^2 + \partial^2/\partial y^2$.
The Hartmann number Ha is defined by $\mathrm{Ha}^2 = B_m^2 D^2\sigma/\rho\nu$. In the experiment, we have that the Hartmann number is always smaller than unity, hence, the last terms in equations (2) and (3) are small, but important since these terms account for the elctromagnetic brake.
The scaling of the variables is defined by $\mathbf{x} = \mathbf{x}^*/D$, $t = t^*/D^2/\nu$, $\mathbf{u} = \mathbf{u}^*/u_0$, $p = p^*/\rho u_0^2$, $\mathbf{j} = \mathbf{j}^*/j_0$, $\mathbf{B}_0 = \mathbf{B}_0^*/B_m$, $\mathbf{b} = \mathbf{b}^*/\mathrm{Rm}B_m, u_o = \nu/D, \mathrm{Rm} = \mu_0\sigma u_0 D$, where the asterisks indicate dimensional variables.
Equation (4) can be decoupled from the previous ones and, hence, the induced magnetic field can be calculated once the velocity field is known. The set of equations (1) to (3) has been integrated using a finite volume code described in references [5] and [4].

3. Results. The flow promoted by the Lorentz force is formed by a central jet that generates two recirculation zones due to mass conservation. This flow was visualized with small spheres coated with silver and illuminated with a horizontal laser light plane at a depth of 3.75 mm from the bottom wall. The resultant pattern is shown in Fig. (2). The circle coincides with the position and size of the magnet. The maximum velocity recorded is approximately 6 mm/s and occurs in the region near the edge of the magnet in the downstream direction as defined by the flow.

The experimental and theoretical streamlines for Re = 30 and Ha = 0.2 are superposed in Fig. (3). The qualitative agreement is satisfactory, indicating that a two dimensional model is adequate to display the salient features of this phenomenon. Velocity fields obtained at smaller distances from the bottom wall display similar features to those shown in Fig. (3) but with smaller magnitudes. The

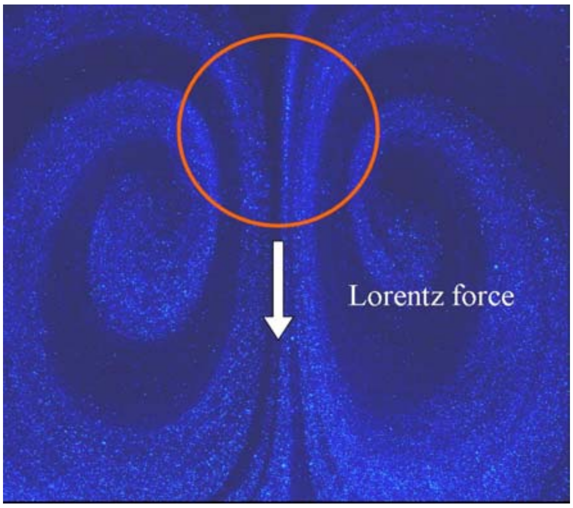

Fig. 2. Photograph of the flow visualized with spherical particles for PIV measurements. The circle indicates the position and size of the magnet.

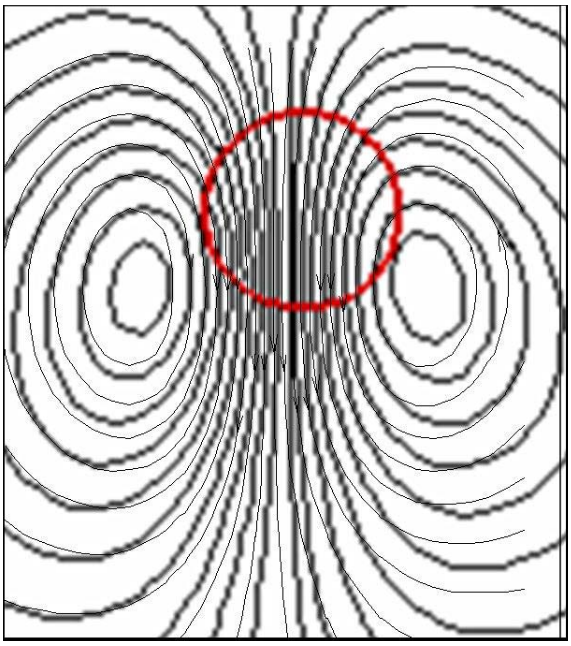

Fig. 3. Experimental (thin lines) and theoretical (thick lines) stream lines for Re = 30, Ha =0.2. The circle indicates the position and size of the magnet.

velocity fields obtained in vertical planes indicate that under the explored conditions there is no transversal motion. It was found that the vortices are elongated in the direction of the Lorentz force when the electric current was increased. Also, the velocity profiles in the transversal plane reveal a clear transition from a parabolic profile in zones, where the magnetic field is negligible, to a Hartmann-like profile near the magnet.

4. Acknowledgments. Financial support from projects No.U41347, CONACyT (Mexico) and IN111705-2, UNAM is gratefully acknowledged.

REFERENCES

1. J. Paret, P. Tabeling. Experimental observation of a two-dimensional inverse energy cascade. *Phys. Rev. Lett.*, vol. 79 (1997), pp. 4162–4165.
2. D. Rothstein, E. Henry, J.P. Gollub. Persistent patterns in transient chaotic fluid mixing. *Nature*, vol. 401 (1999), pp. 770–772.
3. J. Paret, D. Marteau, O. Paireau, P. Tabeling. Are flows electromagnetically forced in thin stratified layer two dimensional?. *Phys. Fluids*, vol. 9 (1997), pp. 3102–3104.
4. M. Griebel, T. Dornseifer, T. Neunhoeffer. *Numerical simulation in fluid dynamics: a practical introduction.* (SIAM, Philadelphia, 1998).
5. S. Cuevas, S. Smolentsev, M. Abdou. On the flow past a magnetic obstacle. *J. Fluid Mech.*, Submitted for publication.