MHD TURBULENT SPECTRA FORMATION IN AN EXTERNAL MAGNETIC FIELD

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It is well known that in the absence of external effects on a turbulent flow, Navier-Stokes (Euler) equation for the second moments in inviscid limit comprises two integrals of motion – energy and helicity. As demonstrated in [1], in the case of a developed turbulence in the inertial and dissipative intervals, the fluxes of these integrals of motion considered as governing parameters lead to combined spectra. The properties of these spectra depend on their ratio in the intervals under consideration. In the case under study, the structure function $D(r)$ in the inertial interval has the form:

$$D(r) \sim \left(\frac{\varepsilon^2}{\eta}\right)^{2/3} \left(\frac{\eta}{\varepsilon}\right)^{\delta} r^{\delta}. \tag{1}$$

Here ε and η are energy and helicity fluxes, and δ depends on the properties of the turbulent flow in dissipative scales.

When a conducting turbulent medium is placed into an external homogeneous magnetic field, its behavior changes depending on the magnitude of this field, since the turbulent flow becomes anisotropic (see, for example, [2]). On the other hand, besides the viscosity, Joule dissipation arises in the system, which introduces attenuation in all scales including the inertial interval.

As follows from experimental and theoretical studies (see, for example, [3]–[5]), rearrangement of the conducting fluid flow depending on the external magnetic field leads to its dimerization in large scales, while in small scales it remains three-dimensional. In our opinion [5], it is connected with the fact that in the presence of nonzero helicity, the flow instability arises with respect to large scale growth in the region

$$\frac{1}{2}k_0\left[1-\left(1-\frac{4\mathrm{N}\cos^2\theta}{k_0^2\nu_H}\right)^{1/2}\right] < k < \frac{1}{2}k_0\left[1+\left(1-\frac{4\mathrm{N}\cos^2\theta}{k_0^2\nu_H}\right)^{1/2}\right] \tag{2}$$

in the Fourier-space, where $k_0 = \tilde{\tau}|C|/\nu_H$, C is the helical part of the velocity field correlator, $\tilde{\tau}$ is the characteristic dissipation time, ν_H is hydrodynamic viscosity (involving turbulent dissipation), N is the Stuart number, $\cos^2\theta = k_z^2/k^2$, and k_z is the wave vector aligned with the magnetic field. Figure 1 clearly shows that with growing N (magnetic field magnitude), the instability region $0 < k < k_0$ is retained for transverse (with respect to the magnetic field) modes and disappears for longitudinal modes.

Experimental studies of the behavior of turbulence arising in a conducting medium under the action of various factors (grate, overflown body, honeycomb, etc.) can be subdivided into two kinds – with the mean velocity $\langle u\rangle = 0$ and $\langle u\rangle \neq 0$. The importance of such subdivision is connected with the following. Besides the energy flux along the spectrum, its mean helicity also plays an important role in the behavior of a turbulent flow.

On the other hand, helicity generation depends on the properties of mean velocity field [6]. As follows from the results of this paper, under a constant external magnetic field, the mean helicity magnitude depends of the mean velocity gradient and vorticity. Hence, even at relatively weak magnetic fields, when the anisotropy they introduce is small, the helicity can play a significant part in the behavior of a turbulent flow.

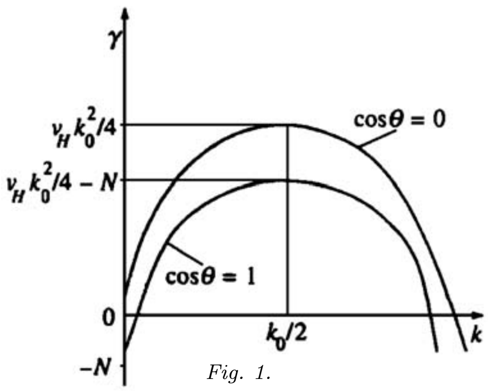

Fig. 1.

In sufficiently weak magnetic fields (at sufficiently low N), when their influence on the Euler equation in the inertial interval can be neglected, energy and helicity fluxes remain governing parameters. Depending on their ratio, the spectrum is specified by Eq. (1).

In very low magnetic fields, when the helicity generation does not play a significant part at the given flow velocity, the spectrum (1) is degenerated into the spectrum

$$E \sim \varepsilon^{2/3} k^{-5/3}. \tag{3}$$

With increasing influence of the magnetic field, two situations are possible. If the mean flow velocity is sufficient to lead to the generation of appreciable helicity in weak magnetic fields, we obtain the following spectrum:

$$E \sim \eta^{2/3} k^{-7/3}. \tag{4}$$

If, however, it is not so, then we are dealing with a mode where the influence of the magnetic field cannot be neglected. In this case, the anisotropy of the flow properties, including turbulent ones, manifests itself. Just this defines the subdivision of experimental results into cases with $\langle u \rangle = 0$ and $\langle u \rangle \neq 0$.

Many experiments were aimed at the study of conducting fluid flows across the external homogeneous magnetic field $\mathbf{B}_0$ (see, for example, [3, 5, 7, 8] and references therein). In these experiments, correlation between turbulent fluctuations of velocity field components aligned with the flow was studied. Practically in all these experiments, the behavior of spectral energy density was as follows. At low Stuart numbers, the spectrum changed with its growth from $k^{-5/3}$ to $k^{-7/3}$. With its further growth, large-scale motion dimerization took place, while small-scale motion remained three-dimensional. In this case, turbulence passed into an intermittent mode with the spectrum $k^{-11/3} - k^{-4}$. It is noteworthy that the quasi-two-dimensional spectrum k^{-3} appeared in experiments (see, e.g., [3]), but it was not limiting in the sense that the system remained three-dimensional at any magnetic field values.

Whereas $k^{-5/3}$ and $k^{-7/3}$ at low Stuart numbers are due to the effect of energy and helicity fluxes, steeper spectra are stipulated by different factors.

To explain the appearance of $k^{-11/3}$, an assumption was made in [7] that in this case the determining parameter in the inertial interval is the super-helicity flux

$$\eta_\omega = \frac{\mathrm{d}(\mathrm{rot}\mathbf{u} \cdot \mathrm{rotrot}\mathbf{u})}{\mathrm{d}t}.$$

However, this conclusion is based only of the dimensionality of η_ω value. Besides, this value is not an integral of motion, and it is rather difficult to understand why it should be conserved, and all other values should depend on it.

On the other hand, with increasing magnetic field intensity, turbulence, as follows from experimental data, tends to an intermittent mode. In this case, in the vicinity of this transition, the properties of energy transfer ε along the spectrum are changed. Why do we revert to the discussion of the behavior of ε ? It is due to the fact that with growing magnetic field the fluctuations across and along the magnetic field can be, apparently, uncoupled, i.e., to the first approximation, they proceed independently, and Joule dissipation does not practically affect transverse modes

We revert again to the case where energy flux in the inertial interval can be a governing parameter (while the helicity flux η is determined by all the three velocity components and cannot be an integral of motion). However, the change in the properties of energy transfer in the vicinity of the transition to intermittency leads to ε dependence on the coordinate and time, i.e. $\varepsilon \Rightarrow \varepsilon(x)$, which was studied in numerous papers starting from [11, 12]. Usually, such an approach is reduced to the choice of a certain spatial region and averaging of $\varepsilon(x)$ over this region. On the other hand, the energy introduced into the system by an external force is independent of the properties of the system in small scales. In case of ε dependence on the coordinates, it seems more logical to us to pass to local characteristics, such as densities of energy flux, and not to integral ones, i.e. to the densities of arriving and dissipating energies.

The density of the arriving energy flux $\varepsilon_V = \mathrm{d}\left\langle U'^2 \right\rangle / \mathrm{d}t\mathrm{d}V$, (where U' – turbulent fluctuations in the force scale) is compensated, under stationary conditions, by the energy flux density $\varepsilon_V = \mathrm{d}\varepsilon(x)/\mathrm{d}V$. In this case, we obtain that ε_V becomes governing parameter. Repeating the arguments produced earlier, we obtain that for the components transverse to the magnetic field in the inertial interval

$$E(k) \propto \varepsilon_V^{2/3} k^{-11/3}.$$

The second case covers experiments described in [9, 10]. In these experiments, turbulence was excited in a conducting medium by a grate drawn vertically along the magnetic field direction. Correlation characteristics of turbulent velocity field were measured along $\mathbf{B}_0$.

As follows from these experiments, spectral properties of the attenuating turbulence depend on the distance from the grate. While in the vicinity of the grate, which had a nonzero channel filling ratio (i.e. possessed nonzero piston properties), a certain mean flow arose, while at a larger distance from the grate the flow was practically absent.

As follows from experimental results at low N (0.6–0.7), in the vicinity of the grate, an appreciable spectral interval with the spectrum close to $k^{-7/3}$ is observed near the grate.

As the distance from the grate increases, both the mean flow (its vorticity) and helicity drastically drop, which leads to the formation of a broad interval with the spectrum $k^{-5/3}$.

Joule dissipation exerts the greatest effect on spectral modes with $\mathbf{k}$ parallel to $\mathbf{B}_0$, which corresponds to longitudinal $\mathbf{B}_0$ components of the correlation function. In this case, it seems logical to choose the parameter $\gamma = \sigma B_0^2/\rho$ in the capacity of a governing parameter, and then the spectral energy density has the form

$$E(k) \propto \gamma^2 k^{-3} .$$

Thus, this spectrum is a result of the increasing effect of the magnetic field on the longitudinal modes of the turbulent field. Similar reasoning was suggested in [9]. Note that "–3"-spectrum is usually identified with turbulence transition into the two-dimensional regime. In fact, "–3"-spectrum connected with enstrophy conservation arises in two-dimensional turbulence similarly to "–7/3"-spectrum connected with helical properties of a turbulent field in three-dimensional turbulence.

However, "–3"-spectrum is present in 3D turbulence connected with longitudinal components only.

Conclusion. In the present work we have examined the behavior of the structural function of homogeneous incompressible turbulence velocity in the presence of an external homogeneous magnetic field.

On the basis of the study of governing parameters determining the behavior of turbulent flow of a conducting fluid at relatively low and high Stuart numbers (such as $\gamma = \sigma B_0^2/\rho$, helicity and energy fluxes, etc.), we have analyzed the properties of spectral functions for longitudinal and transverse (with respect to the magnetic field) components of the velocity field.

Spectra arising at relatively low Stuart numbers are close to helical-Kolmogorov spectra both for longitudinal and transverse components observed in various experiments. With growing magnetic field, flow anisotropy becomes more and more appreciable in large scales, whereas in small scales turbulence remains three-dimensional. However, for longitudinal and transverse modes, a differentiation of governing parameters appears. While for a longitudinal mode Joule dissipation is essential, and respectively, the governing parameter is γ, for transverse ones energy and helicity fluxes are still important. At a still greater increase in the magnetic field, turbulent velocity field becomes intermittent, which is expressed by a considerable dependence of energy flux on the coordinate. In this case, a new governing parameter appears which is connected with local properties of the turbulent velocity field, namely, energy flux per unit mass and volume.

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