TWISTED MHD JET FLOWS

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Introduction. This paper is dedicated to a theoretical consideration of twisted magnetohydrodynamic (MHD) jet flows in cylindrical and spherical coordinate systems. The obtained results may be suitable for astrophysics and MHD technology.

1. Theory. The proposed method is based on using stream functions in the 2D problem under the common approach proposed by E. Shcherbinin [1, 2, 3, 4]. It is suitable for both hydrodynamic and magnetohydrodynamic flows (in a non-induction approximation). The idea of this approach is that in a 2D case it is possible to introduce hydrodynamic stream function related to the flow velocity to satisfy the continuity equation $\mathrm{div}\mathbf{V} = 0$. Magnetic and electric stream functions are introduced in the same way using corresponding Maxwell equations. This method allows to reduce the number of unknown functions from 4 at least to 2 in a hydrodynamic case and from 9 at least to 4 as well as to reduce the number of equations to solve, but these equations are of higher order.

Two coordinate systems that allow in the easiest way to introduce rotation are considered: a cylindrical and a spherical system. Twisting the jet allows for additional possibilities for controlling the jet: new types of magnetic fields can be used additionally along with the usual ones (e.g., see [5, 6]), and changing of the momentum of rotation.

In case of a cylindrical coordinate system, the stream functions are taken in the form:

- for jets propagated along the r-axis (or r-plane): hydrodynamic stream function $\psi = A\, z^{\alpha} f(\eta)$, magnetic stream function $\psi_2 = D\, z^{\alpha} f_2(\eta)$, self-similar variable $\eta = r/z$;
- for jets propagated along the z-axis: hydrodynamic stream function $\psi = A\, r^{\alpha} f(\eta)$, magnetic stream function $\psi_2 = D\, r^{\alpha} f_2(\eta)$, $\eta = z/r$.

In the presence of the electric current, the electric stream function $\psi_1 = C\, z^{\gamma} f_1(\eta)$ in the 1st case and $\psi_1 = C\, r^{\gamma} f_1(\eta)$ in the second case can be introduced. Here A, D, C, α, γ denote coefficients determined by the problem, f, f_1, f_2 are the dimensionless functions.

Some words should be said about the rotational velocity V_{φ}. A standard self-similar approach defines that rotational velocity must be proportional to the $1/z$ in the first case, and to the $1/r$ in the second one, but for the twisted flows this is not correct. If we want to use the condition of conservation of the rotation momentum, we have to take the rotational velocity proportional to the $1/z^2$ and $1/r^2$ respectively. This means that the obtained system of equations is not a self-similar system, but it is possible to separate variables z (or r) and η. Impossibility to use the pure self-similar approach can be explained by the following reasons: the presence of two (in a hydrodynamic case) independent flow characteristics such as the pulse and the momentum of rotation determines both the characteristic length and the characteristic velocity, and this does not allow to obtain a self-similar solution.

To characterize the jet flow, the following quantities can be used: the jet pulse, the momentum of rotation (a pure hydrodynamic case), the magnitude of magnetic field and electric current (a MHD case).

As examples, some classic jet flow problems are considered in the presence of the twist: a twisted Landau jet, a twisted round Schlichting jet and a twisted radial jet.

The solution for the twisted Landau jet problem yields some interesting results. Depending on the parity of pulse and momentum of rotation, the zone of the reverse flow appears near the jet outlet, and the fluid flows around this zone.

In case of a spherical coordinate system, the only possibility to convert the Navier-Stokes and Maxwell partial equations to ordinary equations is to separate variables and choose stream functions as the product of function depending only on R (spherical radius) and function depending only on θ (meridional angle): a hydrodynamic stream function $\psi = F_1(R)\,\Phi_1(\theta)$, all other functions have the same form. Formally, there are two possibilities to separate variables – to make equations depending only on the variable R, and to make equations depending only on the variable θ. But, in fact, only the second case can be implemented.

As in the case of cylindrical co-ordinates, the same characteristics can be used to control the jet: the pulse, the momentum of rotation and magnitudes of electric current and magnetic field.

As examples, a twisted Landau jet, a jet in the round cone and some another problems have been considered.

For the both coordinate systems, possible MHD analogs of the problems under discussion have been formulated and solved.

For rotational coordinate systems, a non-inductive approximation has been created, and also some theoretical investigation under this approximation in a axisymmetric case in other rotational co-ordinate systems has been carried out. The non-inductive approximation means that we take into account the electric currents induced by the interaction of the fluid flow and the magnetic field, but we do not consider magnetic fields induced by these electric currents. The result of this investigation shows that due to different Lame coefficients it is necessary to propose a non-inductive approximation separately for 1) cylindrical, 2) spherical and 3) all other axisymmetric coordinate systems.

2. Conclusions. The results of this work show that the twisting of jet assures additional possibilities for controlling the flow of the fluid in the jet, and allows also to use some new kinds of magnetic fields. It is also shown that it is impossible to obtain common equations in a non-inductive approximation for all axisymmetric co-ordinate systems.

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