SCIN-EFFECT INFLUENCE ON THREE-DIMENSIONAL INSTABILITY OF A TRAVELING MAGNETIC FIELD DRIVEN FLOW IN A CYLINDRICAL CONTAINER

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Three-dimensional instability of axisymmetric time-averaged flows in a cylindrical container driven by a traveling magnetic field is considered. It is assumed that at the container sidewall the vector potential of the mametic field is 2iven by

$$A_r = 0, \quad A_\theta = A_0 \mathrm{e}^{i(\omega t - \widetilde{\alpha} z)}, \quad A_z = 0\,, \tag{1}$$

where $\widetilde{\alpha}$ and ω are the wavenumber and the circular frequency of the traveling magnetic field, respectively. In the case of an infinite cylinder in an infinite TMF inductor the problem for the magnetic and electric fields allows for an analytical solution, which yields the followiniz dimensionless expression for the time-averased electromaenetic force [1]:

$$f_r = \frac{Ft}{\alpha}\frac{Im\big[I_1(\beta^* r) I_0(\beta r)\big]}{\big|\beta I_0(\beta)\big|^2}, \quad f_\theta = 0, \quad f_z = Ft\frac{\big|I_1(\beta r)\big|^2}{\big|\beta I_0(\beta)\big|^2}\,. \tag{2}$$

Here $Ft = B_0^2 \dfrac{\omega\sigma\alpha R^5}{2\rho\nu^2}$, $\alpha = \widetilde{\alpha} R$, $\beta = \sqrt{\alpha^2 + i\gamma}$, $\gamma = \sigma\omega\mu R^2$, and $B_0 = A_0\beta$; σ is the electric conductivity, ν is the the kinematic viscosity, ρ is the density and R is the radius of the container, $I_k(z)$ is a modified Bessel function. More complicated expressions, which take into account the final extent of the inductor and the cylinder and the distance between them, were obtained recently in [2]. The value of dimensionless wavenumber is equal to $\alpha = 2\pi R/L$, where L is the TMF wavelength. At large L the wavenumber tends to zero and the expression for the z-component of the electromagnetic force (2) can be approximated asymptotically as

$$f_z = Ft\frac{r^2}{4} \tag{3}$$

This expression was used for the stability analysis in [3]. In Fig. 1 we compare the force f_z for $\gamma = 1$ and different α using expressions (2) and (3). It is seen that Eq. (3) gives a good approximation only for $\alpha < 1$. Equations (2) must be accounted for already at $\alpha = 2$. For $\alpha > 3$ the force is characterized by a rapid growth near the cylinder wall ($r = 1$), i.e., the well-known skin-effect is observed. Assuming that the TMF wavelength is of the order of the cylinder radius the value of a can be estimated as 2π, and exceeds ten for $L < R/2$. In our calculations we consider $1 \leq \alpha \leq 20$, which allows us to study the influence of the skin-effect on both flow patterns and their stability. For $\alpha < 1$ results of [3] apply. As in [3] we consider the no-slip boundary conditions on all the borders.

The stability diagrams corresponding to the onset of three-dimensional instability with respect to the perturbations represented as $a(r,z)\exp\big[i(k\theta + \lambda t)\big]$ are computed for the aspect ratios of the cylinder $H/R = 1$, 2, 3 and 4. An example

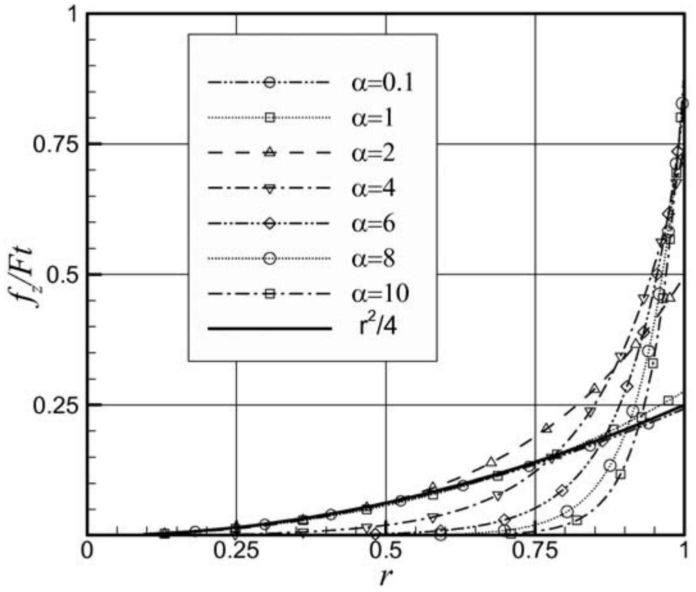

Fig. 1. Comparison of the expressions (2) and (3) for the axial component of the timeaveraged electromagnetic force.

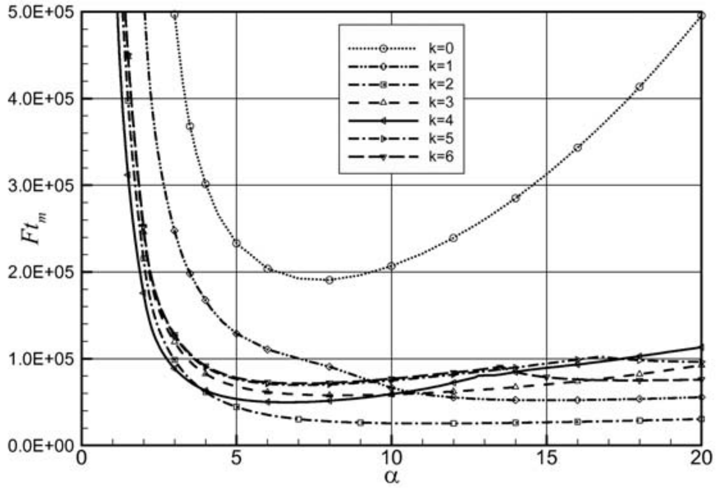

Fig. 2. Marginal stability curves for three-dimensional instability of flows at $H/R = 1$, $\gamma = 1$.

of the stability diagram for $H/R = 1$ is shown in Fig. 2. It is seen that at small values of α, $\alpha \leq 4$, the most unstable mode corresponds to $k = 4$, which is replaced by the $k = 2$ mode at larger α. In general, we observe a steep decrease of the marginal values of Ft with the increase of a from 1 to approximately 6 and a slow increase of Ft_m with the further increase of α.

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