ON MAGNETOHYDRODYNAMIC DRAG REDUCTION AND ITS EFFICIENCY

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Introduction. Various drag reduction techniques were studied numerically and experimentally [1-8, 11-13]. Among others, there is a subtopic of the magnetohydrodynamic (MHD) drag reduction, where the Lorentz force is used for the purpose of drag reduction in an electrically conducting fluid. In recently published papers [6, 7, 8] permanent magnets and high electric current densities are used to achieve reasonable Lorentz forces. This choice, however, usually leads to a low energetic efficiency for the flow of seawater.

We consider a plane channel as the flow configuration. Here the fully developed turbulent channel flow is homogeneous in the streamwise and spanwise directions, thus, periodic boundary conditions can be applied in these directions. This simplifies the numerical solution significantly. We present results of direct numerical simulations of the turbulent channel flow drag reduction using electromagnetic forces. The Lorentz force is created by the interaction of a permanent magnetic field and an electric current from electrodes placed on the bottom wall surface. Two different electromagnetic field cases are considered. At first, an oscillating electric current and a permanent magnetic field create a spanwise oscillating Lorentz force, whereas in the second case a steady streamwise force is created by means of a stationary electric current.

The reason of the low MHD drag reduction efficiency by a spanwise oscillating Lorentz force, as obtained up to now in literature, is explained. The main result of our work is that using a load factor $\kappa \sim 1$ leads to a significant efficiency improvement for all considered cases. We show that the oscillating spanwise Lorentz force reduces the skin-friction drag. The full drag is also reduced, the efficiency is increased by 100 times but is still much less than unity. The application of the streamwise Lorentz force leads to a much more effective drag reduction if we consider the drag as a full force applied to the body. The skin-friction drag increases but the full drag may be reduced to the zero value with a good efficiency [11].

1. Spanwise oscillating Lorentz force.

In the present study we consider a fully developed turbulent channel flow in the presence of crossed magnetic and electric fields. The governing equations for an electrically conducting and incompressible Newtonian fluid in a channel are written in their non-dimensional forms,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla p + \frac{1}{Re}\Delta\mathbf{v} + N\,\mathbf{f}, \qquad \nabla\cdot\mathbf{v} = 0, \tag{1}$$

$$\mathbf{f} = \mathbf{j}\times\mathbf{B}, \tag{2}$$

$$\mathbf{j} = \mathbf{E} + \mathbf{v}\times\mathbf{B} \tag{3}$$

$$\mathbf{E} = -\nabla\Phi_e, \tag{4}$$

$$\Delta\Phi_e = \nabla\cdot(\mathbf{v}\times\mathbf{B}), \tag{5}$$

where $\mathbf{v}$ is the velocity, p is the pressure, $\mathbf{f}$ is the Lorentz force density, $\mathbf{j}$ is the electric current density, $\mathbf{B}$ and $\mathbf{E}$ are themagnetic and electric fields, $Re =$

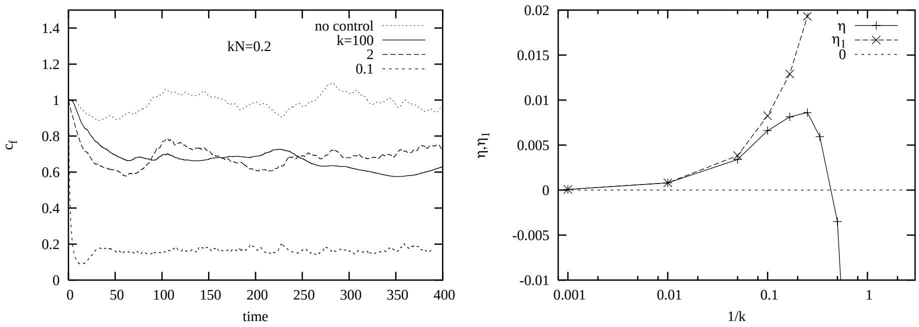

Fig. 1. Normalized drag skin-friction $c_f = C_f/C_f^o$ histories in three various control and no-control $N = 0$ cases (on the left) and the efficiency of full and skin-friction drag reduction versus $1/\kappa$ (on the right).

$v_o d/\nu$ and $N = \sigma B_o d/\rho v_o$ are the Reynolds number and the magnetic interaction parameter. The magnetic Reynolds number $Re_m = \mu_o \sigma v_o d$ is negligibly small in the sea-water case, thus we neglect the induced magnetic field. Following [9], we use a channel half-width d as a length scale, the laminar centreline velocity V_{lam} as a velocity scale v_o, and the other scales are: time d/v_o, magnetic field B_o, electric field $v_o B_o$, electric current density $\sigma v_o B_o$, Lorentz force $f_o = \sigma v_o B_o^2$, where σ and ν are the electric conductivity and the viscosity of the fluid, respectively.

The usual quasistationary electrodynamic approximation $\nabla \cdot \mathbf{j} = 0$ leads to a Poisson equation for the electric potential. In many recent papers the authors consider the electric field as a solution of a Laplace equation. This is true for the vacuum case, but when we find the electric field inside the conducting fluid we may use this solution as an approximation to the true solution only when $\mathbf{E} >> \mathbf{v} \times \mathbf{B}$. We shall show that this case is energetically inefficient.

The numerical simulations were performed using $64 \times 65 \times 64$ grids in a computational domain of $3\pi(L_x) \times 2(L_y) \times \pi(L_z)$, respectively. The third-order Runge-Kutta extension [9, 10] of the spectral method was used. The additional electric potential equation was solved by the same tau-collocation spectral procedure as for the other equations. The Lorentz force was advanced together with the non-linear terms. The Reynolds number is $Re = 3000$. Fig. 1 (left) shows the drag histories in no-control case ($N = 0$) and for three various load factor κ values. The normalized skin-friction drag coefficient is $c_f = C_f/C_f^o$. One can see that in the both cases of $\kappa = 100$ and $\kappa = 2$ the skin-friction drag is reduced approximately by 30%. At a very small load factor $\kappa = 0.1$ the skin-friction drag is reduced by a factor of about 5. In the case of small κ values the Lorentz force acts in a braking way. We may achieve a very small or almost zero skin-friction drag force. But there is no much sense to do this because the force applied to the bottom wall is not small in that case.

Following [7], it is natural to define the efficiency as the ratio between saved and used power

$$\eta = \frac{P_{sav}}{P_{used}} = \frac{C_f^o - C(N)}{C_f^o} \frac{v_\tau^2 \overline{V_{cl}}/v_o^3}{N\overline{E_y \Phi}_{y=-1}}. \tag{6}$$

We consider also the efficiency $\eta_1 = \eta(C_f^o - C_f(N))/(C_f^o - C(N))$ [7], where the electrodynamic part of the drag is extracted.

Fig. 1 (right) shows the both efficiencies η and η_1 versus the inverse load factor $1/\kappa$. Because the full drag coefficient at small κ values is larger than the drag in the no-control case, the efficiency η becomes negative. It means that the MHD control works very inefficiently there. The efficiency has the maximum value $\eta = 8.62E-3$ at $\kappa = 4$, which is about 100 times larger than $\eta = 7.69E-5$ at $\kappa = 1000$ and about 10 times larger than at $\kappa = 100$. This is the effect of decreasing the load factor from very large to reasonably small $\kappa \sim 1$ values. The artificially defined efficiency η_1 may be larger than 1 when κ is small (not shown in the figure). If the electromagnetic force applied to the bottom is not taken into account, the skin-friction drag reduction looks very attractive.

2. Streamwise Lorentz force. The streamwise Lorentz force used in the MHD flow control is an old idea [1, 2]. It seems to us that in the recent papers [5, 6] the existence of an electromagnetic force, which is applied to the bottom of the channel, was not taken into consideration. The flow is accelerated near the bottom and the thrust force applied to the bottom wall is directed against the mean flow stream. This is the case, which we consider here. Both the skin-friction and the full drag coefficients depend on the interaction parameter value. We calculated the flow in the channel for various values of N and then by interpolation found the value $N = N_\star$, at which the full force applied to the bottom wall was equal to zero: $C(N_\star) = 0$. The mechanical power, which should be used to move a flat plate with a velocity $\overline{V}_{cl}$ in the nocontrol case ($N = 0$), is:

$$P_o = (\rho v_o^2 S/2) C_f(0) \overline{V}_{cl}(0).$$

The energetic efficiency is the ratio between P_o and the used electric power, here it is equal to

$$\eta = \frac{C_f(0)\overline{V}_{cl}(0)/v_o}{2N\overline{E_y\Phi}_{y=-1}}. \tag{7}$$

In fact the mechanical power, which should be used in the control case, is larger because the drag coefficient increases:

$$P_{mech} = (\rho v_o^2 S/2) C_f(N_\star) \overline{V}_{cl}(N_\star). \tag{8}$$

It is also interesting to consider another efficiency

$$\eta_2 = \frac{C_f(N_\star)\overline{V}_{cl}(N_\star)/v_o}{2N_\star\overline{E_y\Phi}_{y=-1}}.$$

which shows which part of the power is really used to move the bottom wall (in the frame of reference moving with the centreline velocity V_{cl}). The efficiency η_2 is larger than η since due to the Lorentz force application the skin-friction drag

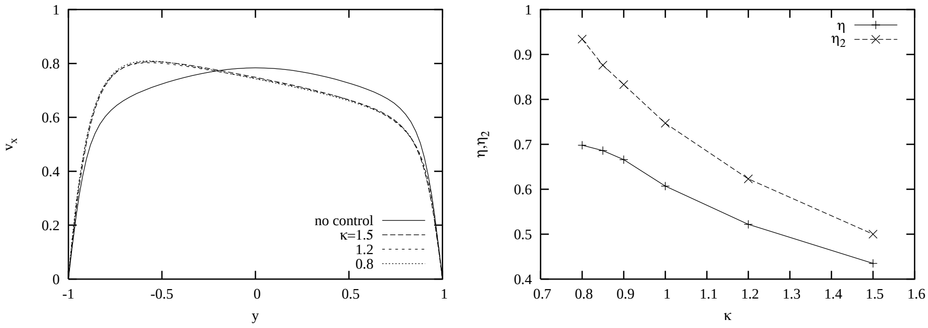

Fig. 2. The average profiles for three κ values and no-control case (on the left). The efficiencies η and η_2 versus the load number κ (on the right).

increases in comparison with the no-control case. Of course, the true efficiency of the applied MHD system is η. Fig. 2 (left) shows the average profile v_x for various κ and for the no-control case, too. All control case profiles are very close to one another and have a slightly larger slope than in the no-control case, which corresponds to the increased skin-friction drag coefficient. Fig. 2 (right) shows the efficiencies η and η_2 versus the load number. It is evident that the efficiency increases for smaller load factors.

3. Flow around a sphere. A gradient-type optimization for several simple magnetic field sources inside a sphere [12] has been performed with the aim to find such distributions of the electromagnetic forces, which may diminish the drag of a sphere. Strong drag reductions have been attained (with factors of up to 15000) by such a flow modification, which leads to an almost counter-balancing of the friction drag with the pressure drag of the bluff body [13]. Only the case of $\kappa \gg 1$ was considered, hence, the possibility of such a drag reduction has been analyzed for a low energetic efficiency up to now.

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