LINEAR ROTATING MAGNETO-CONVECTION WITH ANISOTROPIC DIFFUSIVE COEFFICIENTS

J. Brestensky[1], *T. Soltis*[1], *S. Sevcik*[1]

Dept. of Astronomy, Physics of the Earth and Meteorology, Faculty of Maths Physics and Informatics, Comenius University, Bratislava, Slovakia (Brestensky@fmph.uniba.sk)

Introduction. In geodynamo computer simulations up to date necessary supposition for their tractability was consideration of greater values for viscosity and thermal diffusivity than the values based on transport phenomena at the molecular level in the Earth's core. The smallest diffusive coefficients appropriate for simulations are still two order greater than the corresponding values at the supposed turbulent state of the core. However, the simulation results are surprisingly in good agreement with the geomagnetic field and its secular variations.

Accepting the turbulence in the Earth's core, we have to consider simultaneously the diffusive coefficients of rather tensor quantities than scalar ones, due to turbulent eddies in the core. In meteorology it is necessary to consider turbulent viscosity and thermal diffusivity for the air which are 2-3 orders greater than the molecular values. Furthermore, the values in horizontal directions are negligible in comparison with turbulent diffusivities in vertical direction. It is opposite in surface ocean waters. In the Earth's core at the dominant role of magnetic force, it is more complex and the analogies with ocean or atmosphere are the only good approximations. The approximations are important in order that the complex problem in the first heuristic step of solution could be mathematically tractable.

Investigation of rotating magnetoconvection problems show that at the so-called diffusive instabilities various types of diffusive processes play an important unreplaceable active role. At basic balance of forces, namely, magnetic, Coriolis and buoyancy forces, individual diffusive processes strongly weaken at least one of the above mentioned forces in dependence on running mechanisms. It allows to develop various instabilities of basic state in the framework of many possibilities of competing diffusive processes. In rotating magnetoconvection, the magnetic diffusion weakens the magnetic forces and thermal diffusion and viscosity weaken buoyancy and the Coriolis force, respectively. The greater amount of diffusive processes, the more complex the set of phenomena in the system.

Obviously, it is necessary to consider the influence of the anisotropy of diffusive coefficients in the studies of the rotating magnetoconvection in the Earth's core. Our attention is focused on such types of anisotropy which enable us to find solutions in a separable form. Diffusive coefficients will be isotropic in horizontal directions. There will be a difference in their values in horizontal directions and in the vertical direction. By analogy with surface ocean waters and the lower atmosphere the diffusive coefficients in horizontal directions are much greater or smaller than in the vertical direction, respectively.

The analogy with the atmosphere can be a rough approximation to the results of the Earth's core turbulence study by [3]. The arising turbulence, strongly influenced by magnetic and Coriolis forces, corresponds to eddies of pancake form stretched in the z- and φ-directions (rotation axis and magnetic field) and it is thin in the s-direction (perpendicular to the axis of rotation). It gives, e.g., thermal diffusivities $\kappa_{zz} \sim \kappa_{\varphi\varphi} \gg \kappa_{ss}$ instead of a-anisotropy, $\kappa_{zz} \gg \kappa_{\varphi\varphi} = \kappa_{ss}$, investigated by us. The ocean type o-anisotropy, $\kappa_{zz} \ll \kappa_{\varphi\varphi} = \kappa_{ss}$, corresponds to a stably stratified sublayer of the Earth's core at the boundary with the mantle.

Just instabilities in this sublayer can well correspond to various secular variations, because the fluctuating magnetic fields with shorter time periods and from deeper depths cannot manifest themselves at the Earth's surface.

We compare our anisotropic studies, a- and o-anisotropies, with two isotropic cases – determined and non-determined by turbulence.

1. Model and method of solution. A rotating horizontal fluid layer of thickness d is stratified due to a temperature difference ΔT between the bottom and the top and is permeated by an azimuthal magnetic field linearly growing with a distance s from the vertical z-axis of rotation. Thus the basic state of the considered model is $\mathbf{U}_0 = \mathbf{0}$, $\mathbf{B}_0 = B_M(s/d)\hat{\boldsymbol{\varphi}}$, $T_0 = T_l - (\Delta T/d)(z + d/2)$, where $\mathbf{U}_0$ is the velocity, $\mathbf{B}_0$ is the magnetic field, and $T_0(z)$ is the vertical temperature profile in the layer [1]. The diffusive coefficients are determined or non-determined by turbulence. In the latter case, e.g., anisotropic thermal diffusion is in cylindrical geometry determined by three coefficients κ_{ss}, $\kappa_{\varphi\varphi}$, and κ_{zz} in the sense $\nabla\cdot(\boldsymbol{\kappa}\cdot\tilde{\vartheta}) = \nabla\cdot\left(\kappa_{zz}\partial_z\tilde{\vartheta}\,\hat{\mathbf{z}} + \kappa_{ss}\partial_s\tilde{\vartheta}\,\hat{\mathbf{s}} + \kappa_{\varphi\varphi}s^{-1}\partial_\varphi\tilde{\vartheta}\,\hat{\boldsymbol{\varphi}}\right)$. At isotropic turbulence $\kappa_{ss} = \kappa_{\varphi\varphi} = \kappa_{zz} = \kappa = O(\eta)$ and $\nu_{ss} = \nu_{\varphi\varphi} = \nu_{zz} = \nu$, where η and ν are the magnetic diffusivity and kinematic viscosity coefficients, respectively.

Perturbations are of infinitely small amplitude, therefore, their basic equations can be linearized and in the dimensionless form are

$$\hat{\mathbf{z}}\times\mathbf{u} = -\nabla p + \Lambda\left[(\nabla\times s\hat{\boldsymbol{\varphi}})\times\mathbf{b} + (\nabla\times\mathbf{b})\times s\hat{\boldsymbol{\varphi}}\right] + R\tilde{\vartheta}\hat{\mathbf{z}} + E_z\left[(1-\alpha_v)\partial_{zz} + \alpha_v\nabla^2\right]\mathbf{u}$$

$$\partial_t\mathbf{b} = \nabla\times(\mathbf{u}\times s\hat{\boldsymbol{\varphi}}) + \nabla^2\mathbf{b}, \qquad q_z^{-1}\partial_t\tilde{\vartheta} = \hat{\mathbf{z}}\cdot\mathbf{u} + \left[(1-\alpha_\vartheta)\,\partial_{zz} + \alpha_\vartheta\nabla^2\right]\tilde{\vartheta},$$

$$\nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{b} = 0.$$

The dimensionless parameters used in these equations are (if g, α_T, and Ω_0 are, respectively, acceleration due to gravity, thermal expansion coefficient, and angular velocity): the modified Rayleigh number $R = (g\alpha_T\Delta T d)/(2\Omega_0\kappa_{zz})$, Ekman numbers $E_z = \nu_{zz}/(2\Omega_0 d^2)$, $E_s = \nu_{ss}/(2\Omega_0 d^2)$, the Elsasser number $\Lambda = B_M^2/(2\Omega_0\mu\rho_0\eta)$, Roberts numbers $q_z = \kappa_{zz}/\eta$, $q_s = \kappa_{ss}/\eta$, anisotropic parameters $\alpha_\vartheta = \kappa_{ss}/\kappa_{zz} = q_s/q_z$, $\alpha_v = \nu_{ss}/\nu_{zz} = E_s/E_z$. In a- and o-anisotropies we have horizontal isotropies, i.e., $\kappa_{ss} = \kappa_{\varphi\varphi}$ and $\nu_{ss} = \nu_{\varphi\varphi}$.

It is reasonable in MAC modes study to distinguish cases when only thermal diffusivity is anisotropic ($\alpha_v = 1$ and $\alpha_\vartheta \neq 1$), and when both viscosity and thermal diffusivity are anisotropic ($\alpha_v \neq 1$ and $\alpha_\vartheta \neq 1$). The high electric conductivity of the Earth's core makes the magnetic diffusivity isotropic in all investigated cases.

Velocity and magnetic field perturbations, $\mathbf{u}$ and $\mathbf{b}$, are split into poloidal and toroidal parts $\mathbf{u} = k^{-2}[\nabla\times(\nabla\times\tilde{w}\hat{\mathbf{z}}) + \nabla\times\tilde{\omega}\hat{\mathbf{z}}]$ and $\mathbf{b} = k^{-2}[\nabla\times(\nabla\times\tilde{b}\hat{\mathbf{z}}) + \nabla\times\tilde{j}\hat{\mathbf{z}}]$ with $\tilde{w},\tilde{\omega}$ representing the perturbation $\mathbf{u}$, $\tilde{b},\tilde{j}$ the perturbation $\mathbf{b}$, and the temperature perturbation $\tilde{\vartheta}$. All perturbations ($\tilde{w},\tilde{\omega},\tilde{b},\tilde{j}$ and $\tilde{\vartheta}$) have the form of waves propagating in azimuthal direction and warranting the separability of solutions $\tilde{f}(z,s,\varphi,t) = \Re e[f(z)J_m(ks)\exp(im\varphi + \lambda t)]$, where $f(z) = w(z)$, $\omega(z)$, $b(z)$, $j(z)$, $\vartheta(z)$, and $J_m(ks)$ is the Bessel function of the 1st kind. For the functions $f(z)$ we have the set of ordinary differential equations, which is reformulated to eigenvalue problem with the Rayleigh number, R, as an eigenvalue. We consider mechanically the boundary stress free and electromagnetically perfect conductors.

2. Numerical results and conclusions. We present numerical results in four subfigures in Fig. 1, firstly three sets of graphs (a), (c), (d) for critical numbers R_c, k_c and σ_c versus the Elsasser number Λ for four cases of anisotropies or isotropies in each figure (o- and a- anisotropy, and isotropies with the Roberts number $q \sim 1$ and $q \ll 1$, respectively). In these three sets of graphs only a single azimuthal wave number, $m = 1$, is with single vertical wave number $l = 1$. In the 4th set of graphs (b) R_c vs Λ is for various azimuthal wave numbers, $m = 1$, 2, 5 and 30, in the case of o-anisotropy. In all figures the curves $R_c(\Lambda)$, $k_c(\Lambda)$ and $\sigma_c(\Lambda)$ correspond to the preferred modes (with exception of MC modes with Rayleigh numbers, $R = 0$, which with corresponding k and σ are not critical R_c, k_c and σ_c, in the o-anisotropy case for $\Lambda \gtrsim 2000$). Discontinuities in the curves (in the 0th, 1st and 2nd derivation with respect to Λ) just correspond to the changes of preference among competing modes.

At the lower Λ there is an obvious transition of preference between a "viscous mode" and a hydromagnetic, mode which becomes the preferred mode for greater Λ. (The name"viscous modes" is related to their non-existence in inviscid fluid.) It holds $R_c(\Lambda) \simeq c_R$, $k_c(\Lambda) \simeq c_k$ and $\sigma_c \simeq c_\sigma \Lambda$ for viscous modes, where the coefficients c_R, c_k and c_σ are independent on Λ, but are variously dependent on other parameters, e.g., Ekman and Roberts numbers or anisotropy parameters, α_v and α_ϑ, represented by α if $\alpha_v = \alpha_\vartheta$.

The mechanisms of the hydromagnetic modes development are various and with the magnetic force being important they (in particular, diffusive instabilities) sensitively depend on diffusive processes, and consequently on various cases of diffusive coefficient anisotropy. We can easily distinguish thermal modes and magnetically driven modes, e.g., MAC and MC modes at $\Lambda \gtrsim 0.3$ and $\Lambda \gtrsim 2000$, respectively, for isotropic cases $q \sim 1$. Due to the importance of additional magnetic diffusion at hydromagnetic modes developement, all curves in the graphs are more

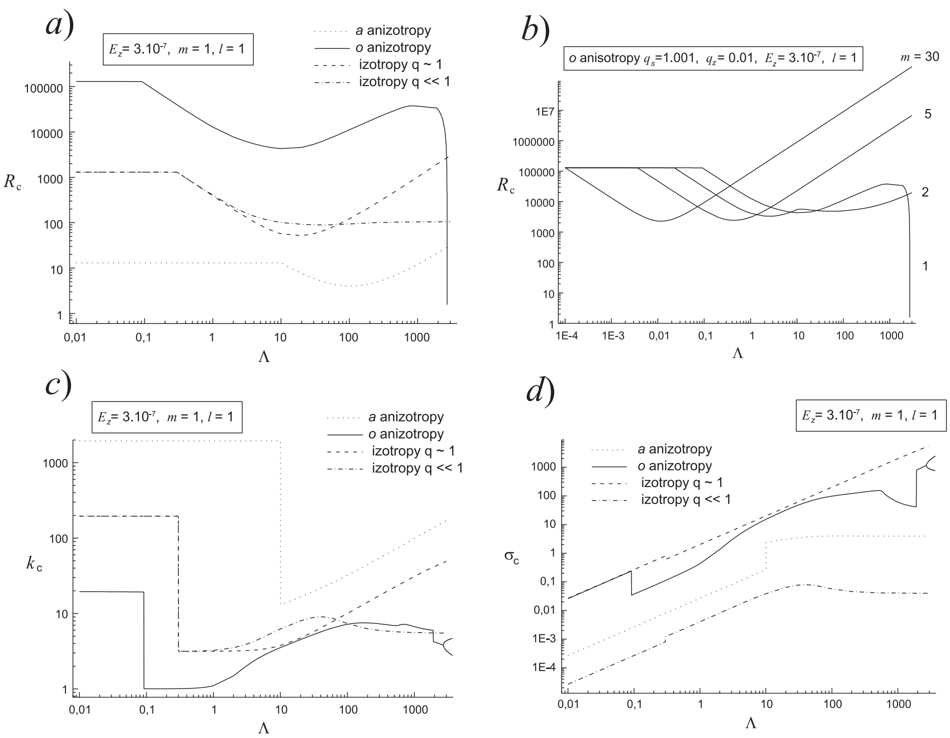

Fig. 1. (a), (c), (d) Critical numbers R_c, k_c and σ_c versus the Elsasser number, Λ, for various cases of anisotropy and isotropy of diffusive coefficients; (a) Rayleigh number, R_c, (c) radial wave number, k_c, (d) frequency, σ_c; (b) critical Rayleigh number, R_c, versus Λ for various azimuthal wave numbers, $m = 1$, 2, 5 and 30 in the o-anisotropy case of diffusive coefficients.

complex for greater Λ. Transition from the "viscous mode" to the hydromagnetic mode at Λ being dependent sensitively on the anisotropy parameter α and the azimuthal wave number m is manifested by jumps in values of k_c and σ_c, and by the change of behaviour quality for $R_c(\Lambda)$, i.e., from the constant R_c value of the "viscous mode" to the curve of parabolic shape.

Referring to the dependence of the modes on anisotropy, it is evident from the graphs that *a*-anisotropy facilitates the convection because it decreases the values of R_c both for viscous and for hydromagnetic modes. Further, the greater a-anisotropy, i.e., $\alpha \ll 1$, the smaller R_c value. Similarly, a-anisotropy increases k_c values and more intensively for a greater a-anisotropy, i.e., it makes the convection cells shorter in the s-direstion. Likewise, the greater a-anisotropy causes an increase of the preference region of viscous modes to greater Λ, thus with transition from viscous to hydromagnetic modes at greater Λ. In contrast to the a-anisotropy, the o-anisotropy embarrasses the convection because the R_c values are greater for a greater o-anisotropy ($\alpha \gg 1$). Critical radial wave numbers, k_c, decrease with the increase of the o-anisotropy; the convection cells are greater in the s-direction. The transition of viscous into hydromagnetic mode is shifted to smaller Λ for the greater o-anisotropy. The case of o-anisotropy gives two obvious examples of transition between various hydromagnetic modes at greater $\Lambda = O(1000)$. First, there is a transition at $\Lambda \sim 500$ into a mode with smaller σ_c. Second, transition at $\Lambda \sim 2000$, which is not in the *a*-anisotropy, is the transition of MAC into the MC mode. The Rayleigh number, R_c, of the MAC mode at $\Lambda \sim 2000$ strongly decreases with Λ, reaching negative values only for $m = 1$. Thus $R_c = 0$ is the beginning of the MC mode existence, which do exist at zero Rayleigh number, $R = 0$, for greater amount (but limited) of vertical wave numbers, l. These MC modes at the o-anisotropy, propagating only westwards, are the diffusive instabilities similar to the MC(W) wave type instabilities [2] in the turbulent isotropic case (of Roberts number $q \sim 1$). However, in the o-anisotropy case they need no too high values of the Ekman number, $E = O(10^{-3})$, and the anisotropic Ekman number values are $E_z = O(10^{-7})$ and $E_s = O(10^{-5})$, which are much closer to the realistic Earth's core Ekman number, $E = O(10^{-9})$, based on turbulent viscosity. Despite the fact that the existence condition of the MC(W) modes requires greater Λ for smaller Ekman numbers as well as in the isotropic case, in the o-anisotropy case the values of Λ are again more realistic, e.g., the MC(W) modes arise at $\Lambda \sim 100$ and $E_s \sim 10^2 E_z \sim 10^{-3}$ in comparison with the isotropic case, where $E\Lambda \gtrsim 1$ with e.g. $E = O(10^{-3})$ and $\Lambda = O(10^3)$. Further the MAC mode bifurcates into two MC(W) modes as is evident from Fig. 1*c*,*d*.

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